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## **Foundations to the Unified Psycho-Cognitive Engine**

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# Foundations to the Unified Psycho-Cognitive Engine

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## Abstract

This document outlines the key features of the SNL psychological engine. The engine is designed to be a generic presentation of cognitive entities interacting among themselves and with the external world. The engine combines the most accepted theories of behavioral psychology with those of behavioral economics to produce a unified simulation of human response from stimuli through executed behavior. The engine explicitly recognizes emotive and reasoned contributions to behavior and simulates the dynamics associated with cue processing, learning, and choice selection. Most importantly, the model parameterization can come from available media or survey information, as well subject-matter-expert information. The framework design allows the use of uncertainty quantification and sensitivity analysis to manage confidence in using the analysis results for intervention decisions.



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# 1. Introduction

The purpose of this project is to develop a decision-support system for national security decision makers who must address both kinetic and non-kinetic interventions. This work develops a computational framework for analyzing the behaviors of individuals and populations, over time, in response to information operations, diplomacy, and other intercessions. Building off of Sandia National Laboratories' (SNL) Cognitive Science expertise, we have developed a data-driven, analytical cognitive framework, anchored to a self-consistent psychological foundation. Predicting human behavior is notoriously wrought with uncertainty. However, all time-critical decisions necessarily take place in environments of uncertainty. The issue is to understand the uncertainty and to establish the level of confidence that the analysis results support. Because of its responsibility to ensure the U.S nuclear arsenal is both safe and reliable, SNL has become the premier laboratory for uncertainty quantification, risk assessment, and verification & validation. This same responsibility has made integrated system engineering and complex systems-of systems analysis a SNL priority. We have combined the psychological-modeling with confidence-management methods to provide a reliable process for assessing the multifaceted and shifting interactions of intervention options. We use the concept of "intervention" rather than "course of action" because we only emphasize those endeavors that intervene to affect the behaviors of the system and individuals of interest.

The basis for the computation system is a synthesis of experimental-data-supported psychological theories of human behavior. This synthesis is further supported by an independent assessment of theory-based analytical studies of historical socioeconomic data. We have integrated that unique set of elements from psychological theory that are consistent with economic theory, experimental data, and historical data on human behavior. We have developed a unified framework that connects the multiple scales (from individual to societal interactions) of human behavior to the external (geopolitical, physical, and socioeconomic) world. Human behavior reacts to the local perceptions of people's actions and to world conditions in a feedback process that cause behaviors, conditions, and events to unfold over time. Our analyses emphasize these response and counter-response progressions, whose recognition can prevent blind-siding and counterproductive interventions.

The new framework appears to comprehensively describe the process of human behavior, inclusive of cultural, biological, and institutional constraints and conditions. The framework is based on first principles that can encompass an unlimited number of entities with any number of alternative decisions, and with any level of interrelationship complexity. Because we only allowed the use of theories that 1) were mutually self consistent, 2) would integrate into a complete representation of behavior from stimuli through to action, 3) would translate to a unique set of computational equations, and 4) could be instantiated, tested, and verified using accessible data, we can 1) readily use available data on individual or regions to calibrate the model, 2) use Subject

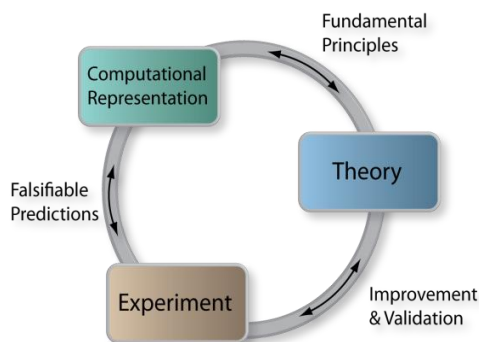
Matter Expert (SME) data to augment data sparsity, 3) test hypotheses about alternative interventions and behavioral responses, 4) quantify the uncertainty (risk) that an intervention will produce the desired results, and 5) follow the time-dependent consequential counter-responses from an intervention. Most importantly, the framework naturally captures the implications of new (even unique) information flows such as may be considered in information operations or other interventions.

This document gives an overview of the framework logic and its function.



## 2. Conceptual Foundations

Our hypothesis is that human behavior can be modeled. Specifically, we assert that essential human behaviors can be computationally modeled based on well vetted psycho-social and economic theories. These models can capture cultural differences and individual uniqueness. The models capture the collective knowledge of subject matter experts and incorporate all available information regarding individuals and their environment. We utilize the approach depicted in Figure 1 to ensure the quality, applicability, and legitimacy of analysis results. Data, theory, and computation results must all be mutually consistent. Self-consistency alone is not sufficient to ensure valid assessments but without self-consistency, an assessment is certainly invalid.

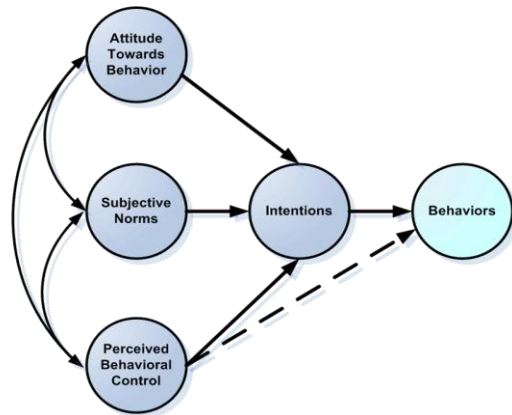


**Figure 1: Model-Theory-Data Approach to Analysis**

The analysis framework is founded on established psychological, social, cultural and economic models. These models contain testable hypotheses that have been extensively evaluated with experimental and historical data. The primary psychological and social theories include:

Theory of Planned Behavior (Ajzen 1985, 1991, 2005; Armitage & Conner 2001)  
Expectancy Value (Fishbein 1961, 1962, 1963, 1975)  
Elaboration Likelihood (Petty & Wegener 1981, Petty & Cacioppo 1986, 1999)  
Cognitive Dissonance (Festinger 1956, 1957)  
Social Learning (Rotter 1945, Bandura 1977)

As an example, Figure 1 portrays the basic elements of The Theory of Planned behaviors. Norms, attitudes, and internal controls interact to form intentions that may become a realized behavior. Background references to the above theories are noted



**Figure 2: Theory of Planned Behaviors (Ajzen, 1991)**

In parallel, a set of economic theories, also extensively evaluated with experimental and historical data, perfectly mesh with the selected psychological theories. Economics is simply people making choices. There is abundant time-series data on economic decisions, across cultures, that must completely overlap with the psychological view of those same decisions. These economic theories include:

Bounded Rationality (Simon 1957)

Qualitative Choice (McFadden 1984, 2000)

Imperfect Information (Stiglitz 1985, 1986, 2002)

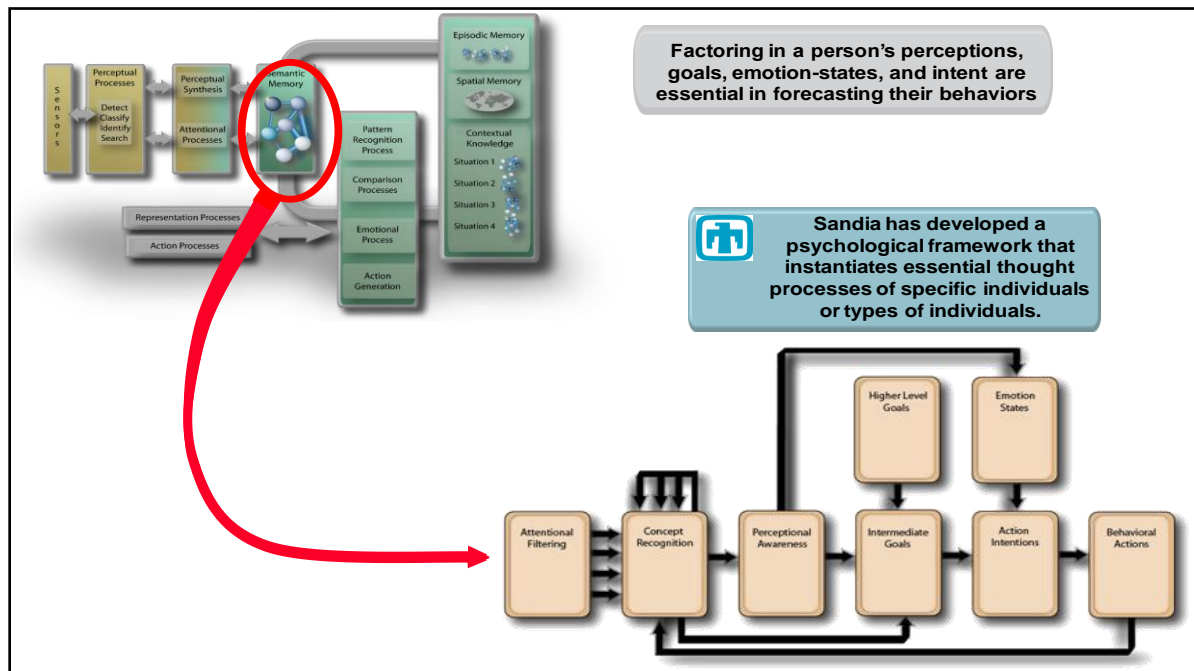
Risk Asymmetry (Tversky & Kahneman 1971, 1972, 1973a, 1973b, 1973c, 1974, 1979, 1979a, 1979b, 1981, 1982, 1984, 1986, 1990, 1993a, 1993b, 1996, 1998, 1999, 2000, 2003)

Stock & Flow Cointegration (Granger & Engel 1981, 1987)

Figure 3 depicts the comprehensive logic of Sandia High-Fidelity Cognitive Modeling program. This effort has a goal of modeling individual on a neurological basis with high fidelity. The work described here emphasizes the highlighted “behavioral” component of this program that is most relevant to national security interventions. This work was originally derived as the Sandia Human Embodiment and Representation Cognitive Architecture or SHERCA (Bernard et al. 2006, 2005 a, 2005b, 2005c).

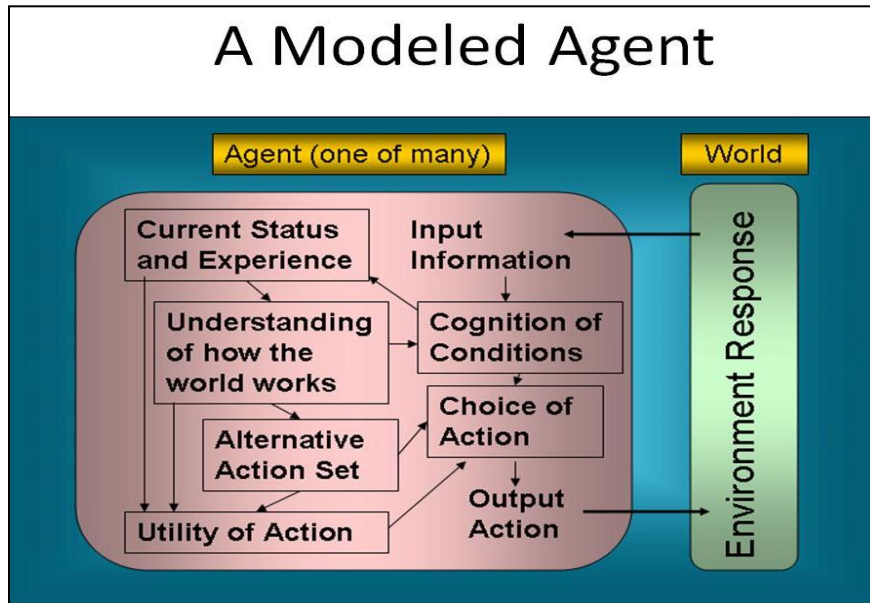
The psychological engine takes the sensor, perceptual process, perception synthesis and the definitions of the attentional processes, representation process, and action process noted in Figure 3 as “hard-wired. How notions select cues to make a pattern of information affecting the selection of choices and producing behavior is defined via a fixed blueprint (discussed in the next section). Although, the semantic memory and Contextual Knowledge portions of Figure 3 are defined with the blueprint, the actual dynamics are explicitly simulated in the model. The model does not deal with spatial memory or the pattern recognition process. It does fully simulate episodic memory, comparison process, and emotional processes.

Specific to the Episodic Memory portion emphasized in the psychological model, the Notion Formation piece of the psychological engine has an exact correspondence with Attentional Filtering, Concept Recognition and Perceptual Awareness. The attitude and cognitive resource aspects of the engine emulates Higher Level Goals and Intermediate Goals. The Notion Formation and Cognitive Resource calculations produce the Emotion States due to current circumstances or condition sensitivities, respectively. The strongest part of the modeling describes the dynamics of Action Intentions and Behavioral Actions.



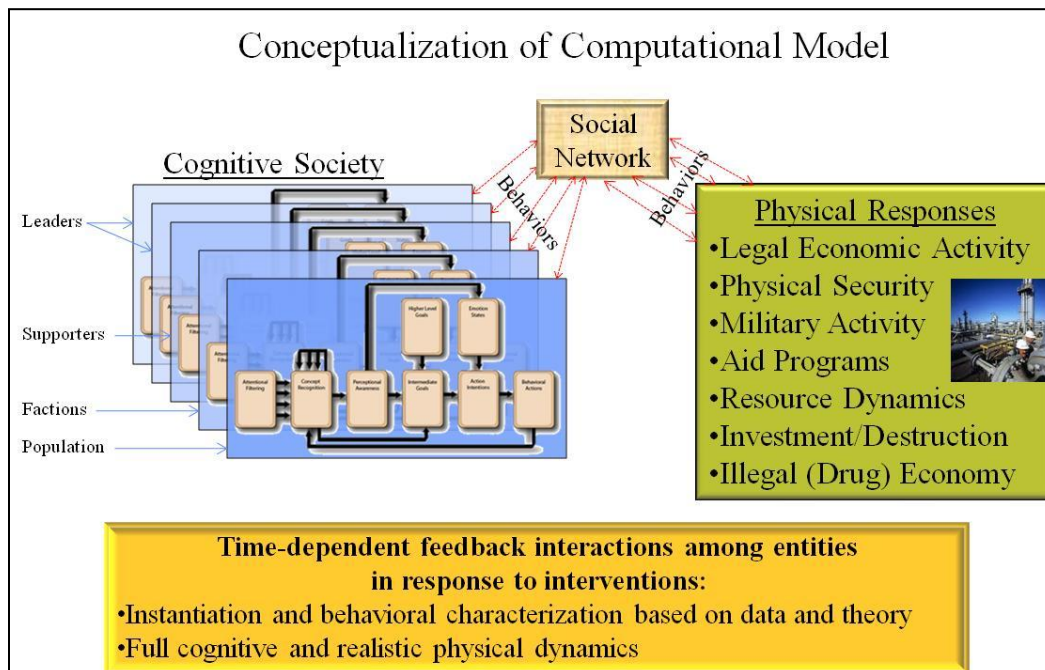
**Figure 3: Sandia High-Fidelity Cognitive Modeling A Modeled Entity**

From an economic perspective, individuals are routinely simulated using agent-based modeling. Figure 4 show a generic representation of behaviors and interaction logic. Individual cognition is always embedded in a feedback environment. Agent based models typically use relatively simple rule-based calculations of behaviors. Our framework replaces the simple rules with a realistic characterization of the actual behavioral processes. The mathematical representation of these processes is largely a re-application of the Backus and Glass (2006) to match the specific needs of influence operations.



**Figure 4: Agent-Based Modeling**

The combined logic of Figure 3 and Figure 4 naturally lead to the structural logic of our overall framework as illustrated in Figure 5. The economic theory provides a means to transform a cognitive entity into representing an individual, a group, or a society. Individuals interact with each other through social interactions and through the physical consequences of human behavior. These physical consequences are the economy itself, but are also the institutional and geopolitical aspects of it.



**Figure 5: Structural Overview of the Unified Psychological Model**

### 3. Modeling Methodology

To model the consequence of interventions, it is necessary to not only model the initial behaviors of affected individuals, but to also determine how interactions with other individuals and the physical world, over time, can alter the outcome. The changes over time are called dynamics. The feedback processes among individuals and the physical world unfold dynamically and cause the outcome of an intervention to, for example, start off going in the desired direction, but in the long term lead to counter-responses that generate new concerns without improving the original issue. The delay between behaviors and impacts can cause secondary dynamics that make it extremely difficult to know whether the ups and downs of behavioral responses and counter-responses will ultimately lead to the desired outcome.

The computational modeling of national security interventions needs to address the dynamic evolution of the integrated socioeconomic and geopolitical system. Such systems are most readily modeled using differential equations. Differential equations not only simulate the dynamics, but additionally they causally describe why the dynamics occur. The System Dynamics methodology developed at MIT is commonly used to model social systems whose interactions are expressible with differential equations (Sterman 1994, 2000).

The process for developing a psychological model using the system dynamics methodology starts with a description of the psychological theories the model must simulate. These theories need to encompass all the salient considerations needed to make a comprehensive system's model describing the problems of interest. Note that there is no attempt of model the entire system, but only those aspects of the system relevant to the problems to be addressed/analyses. The next step is to develop a causal-loop diagram the causally relates all the interactions embodied in the theories. The casual loop diagram is next mapped to a stock-and-flow diagram that explicitly details the flow of information and physical quantities through the system.

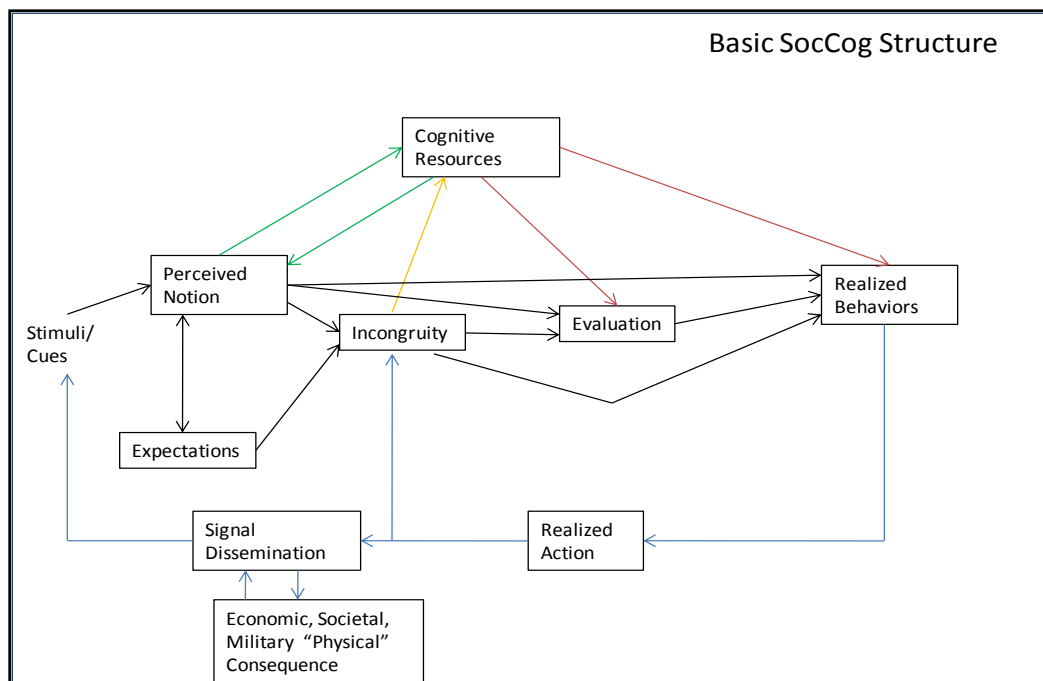
A key feature is the designation of stocks that represent the accumulation of information, experience, monetary, or physical quantities. These stocks are called "state variables" and they largely characterize the nature of the system and its responses. The difference in the value of stocks over time increments is the "differential" part of the differential-equation approach to computational modeling.

The exact mathematical expression of the theory is anchored in the accumulation of flow into and out of the stocks. The mathematical expression of the flows comes from a causal interpretation of the theory into the language of mathematics. The key equations will be described later in this report. Only those theories that have a measurable meaning, supportable, a least in principle, by historical or experimental data, are included in the model. The data determines the parameters that control the progression of the simulated values through time. Rigorous statistical techniques

determine the appropriate parameters and the uncertainty associated with their use. This uncertainty can later define the confidence in the results of an intervention analysis.

## 4. Computational Foundations

At a computational level, the cognitive entities use information in the manner illustrated in Figure 6. Stimuli are the physical realization of world conditions and of human action. When an individual places these stimuli in context, they become cues that inform or affect behaviors. The grouping of cues forms a pattern. For example, the observation of asphalt, cars, sidewalks, and buildings act as cues, giving you the notion that you are on a city street. We use the term “notion” rather than “perception” because the term “perception” can often denote a higher level of cognition than the recognition of simple physical stimuli, such as, the higher-level perception that quantum mechanics better explain atomic phenomena than thermodynamics and opposed to primal sensation of “that pin is sharp!” Notions typically take on importance when they are incongruous with (different from) expectations. Expectations are often the memory of the status-quo or the anticipation of future conditions. Cognitive resources are our learned attitude toward a condition (the condition being a perceived notion or incongruity) or our learned ability to respond to a condition. Our cognitive resources and perceptions of a situation (via notions and incongruities) act together to help us evaluate the choices we have to respond to those conditions. The result represents our intentions. The execution of those intentions further depends on the level of the incongruity and our attitudes toward that behavior. Once we initiate a behavior, it takes time before it becomes an action affecting the external world (including other individuals). Depending on the proximity or our social network, the realized consequence of our actions becomes the cues to some individuals but not to others.



**Figure 6: Computation Elements of the Behavioral model.**

The feedback logic of one entity's behavior becoming another entity's stimuli (cues), possibly through the intermediation of external physical processes, explicitly captures the social network considerations that are often the domain of more-abstract agent-based modeling. An entity is an individual or a group.

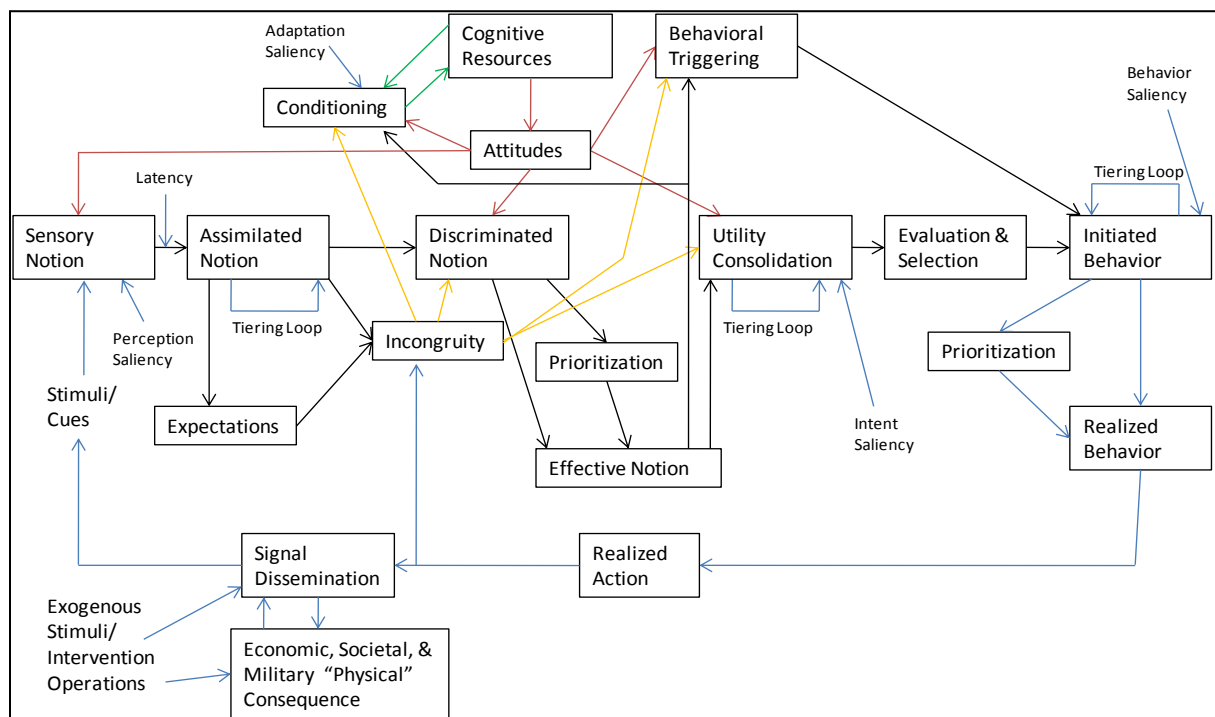
The approach for this modeling is made possible by assuming a fixed set of potential behaviors embodied in a representation of the individual. The representation contains the preferences and personality characteristics pertinent to the relevant decision-making. It is called the "blueprint" and it fully characterizes a specific individual or group of individuals. While the magnitude of interactions may change, the model does not produce new paths of cognition. All potential interactions are determined via initial parameterization of the model. Over the time, frame of the model simulation (at most a couple of years and often on the order of weeks), there should be little possibility, and there is little predictive capability for modeling, that entities would change their behaviors outside the domain of their historical experience and habits. (See Appendix 15 for an expanded discussion on relaxing the assumptions of a "blueprint" approach.)

The mathematical expression of what stimuli cause cues and what choices or behaviors those cues can invoke has to be determined a priori through the use of subject matter experts (SMEs) and available data. SMEs can hypothesize notions and perception that are not reflected in the data. Analytical methods can allow an estimate of how those hypothesized behaviors could occur based on the knowledge of an individual's behaviors in other circumstances. The singular personality of an individual has a large affect on all his or her decisions. Uncertainty analysis could determine the potential for such a behavior to affect the policy selection of external security interventions. (The assessment of these interventions is the actual purpose of the model, not the pin-point prediction of individual behavior.) The model cannot generate potential behaviors that are beyond the imagination of SME's and are not reflected in available data. These unknown unknowns are a limitation in all realms of physical and social science. Nonetheless, the "blueprint" for an individual within the model is the best representation of that individual's behavior characteristics available. Any other representations would be less valid and less reliable.

Figure 7 shows an exploded view of model structure of Figure 6. Each block in the diagram contains only one to two equations. Each of these equations has a simple theoretical construct that will be discussed briefly below and fully elucidated in the Appendices. Note however that each block can process large flows of data. There are typically a large number of stimuli generating a large number of notions, leading to a large number of potential choices and behaviors, across a large number of individuals. Some of the differences to note between Figure 6 and Figure 7 are the decomposition of the "Perceived Notion" in Figure 6 into several subcomponents in Figure 7, such as the sensory and assimilated notion. It takes time to cognitively recognize a set of cues. Cues can also produce emotive notions that characteristically occur faster than cognitive notions and use minimal information. The emotive notions can set the "mood" for processing the cognitive



information, often adding a risk aversion element to the choice invoked by the cognitive information (Hertel 2000, Bless 1996, Lerner 2010, Forgas 1995, Pietomanoco 1987). Research shows that emotive and non-motive components are both part of the normal processing that leads to behavior (Zajonc 1980, 1984; Martin 1993; Wagar 2004, Slovic 2005). The model explicitly recognizes and uses both these categories of information flow.



**Figure 7: Detailed Model Cognitive components.**

Figure 7 contains what are noted as “Tying Loops.” Specific notions (such as you realizing there is a fire in your house), can dramatically amplify your realization of other notion/cues such as the location of doors and other occupants of the house. Similarly, making one decision may affect your selection of a related decision. The same is true for executing behaviors. Attitudes affect the importance you may place on information. Attitudes are explicitly calculated in the model and are based on cognitive resources (experiences, abilities, and beliefs). Learning is noted as conditioning in the model and is an effort to reduce an incongruity by developing the ability to accommodate or effectively respond in the presence of a notion. Attitudes, emotive content, and cognitive information all act to determine the utility of a choice. These utilities come together to shape the probability of making a specific choice. Limitations in mental processing and physical response mean the individual must prioritize notions and behaviors when either becomes potentially excessive (Gigerenzer 1996, Dolan 2002). For example, changing the radio station when you hear a song you dislike is quickly neglected when you see the car ahead of you hit another car.

Figure 7 depicts the psychological components that interact, feedback, and combine to produce behavior. People are constantly exposed to a large number of stimuli. They attempt to find patterns in these stimuli to help predict their environments. Only a small fraction of stimuli can be processed and recognized as relevant cues for prediction. A specific pattern of these cues can produce a notion of the current environment. In the model, relevant cues include political, social, physical and economic conditions. The inflation rate, for example, is an economic cue that may lead to a notion about the health of the economy.

Perceptions combine with beliefs (attitudes) and expectations to demarcate the motivations (preferences) and affects (emotive contexts) people use to weigh alternative responses to stimuli. Competing motivations lead to higher probabilities to select some responses and lower probabilities to select others. The selection of a potential response is also often denoted as the intent. As noted previously, the execution of the intent is the behavior.

Beliefs and expectations are general terms to describe accumulated experiences and memories, influenced by genetic and physiological constraints, which give context to cues, motivations and behaviors. Beliefs and expectation are internal references consisting of anything learned that an entity brings into the decision-making process. These include knowledge, attitudes, norms, behavioral controls, emotional context, and intuition. These factors bring the decision-maker's specific characteristics and life experiences (even those not related to the decision being made) into the decision-making process. The facets of belief directly amplify or diminish the processes of perception, motivation, and behavior. Perceptions and Behavior can in return affect beliefs and expectations.

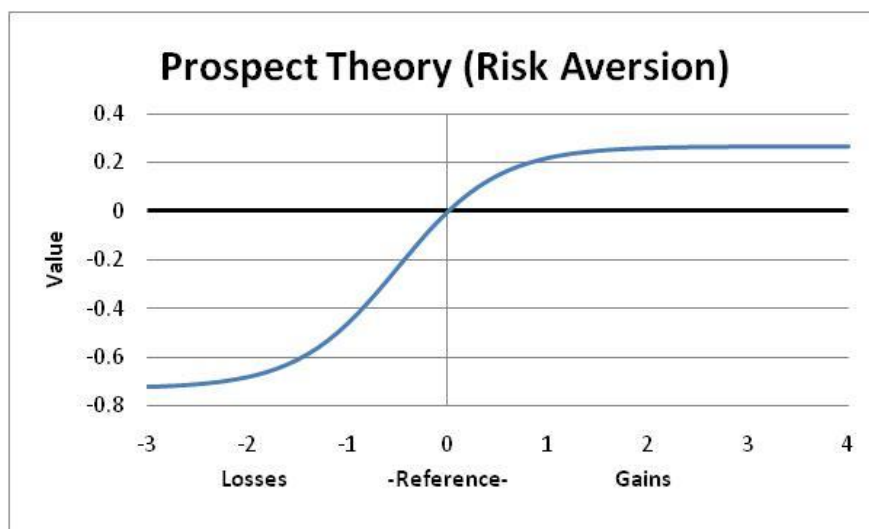
When a current notion rises to full consciousness, it is compared to an expectation that is stored in long-term memory. Expectations serve to categorize, classify, and structure a person's notion of the world. We have expectations of ourselves, of others, and of the world around us. The model tracks each cognitive entity's expectation for each potential notion. People compare notions and expectations in order to assess the current state of themselves and their environments. If there is a discrepancy between notions and expectations, the cognitive entity experiences incongruity. Incongruity is a measure of the difference between what is perceived and what is expected. If incongruity is high enough, the decision-maker will have a negative affective reaction that will cause them to try to reduce this incongruity, by changing their beliefs (e.g., attitudes), or changing the (relative importance) weighting of motivations, notions, or behaviors. Behaviors may reduce the incongruity by bringing the external environment in concert with the (modified) expectations.

The model includes the set of choices (intents) that the decision-maker may choose based on perception, incongruity, and cognitive resources. To determine which intent the decision-maker prefers, the model first calculates the utility, or potential benefit, of each possible choice. These are

reflected in the motivations and affects. Once the utilities of all potential intents are determined, the model assigns a probability to each using the multinomial logit function, a well-established function used in qualitative choice theory (Ben-Akiva and Lerman 1985; Train 1986; Train 2009). The decision-maker's utility of an intent or the strength of a behavior is amplified or dampened based on dissonance and internal references.

Moods are essentially lingering emotive notions. Altering conditions can cause moods to change over time, but typically not instantaneously. Therefore, decisions based solely on objective information may be different than those made in the presence of a specific mood (Rusting 1998, Mellers 2001). Because moods can arise quickly, decisions later in time, when moods have subsided, may be significantly different from those made when the individual is in a highly emotional state (Tiedens 2001).

The terms Saliency and Latency in Figure 7 note the parameterizations that capture the importance the individuals place on information (facts or feelings). The individual can adapt the importance he/she places on information as a result of conditioning and modified cognitive resources (Schwarz 1996). This adaptation reflects itself as strengthened or weakened attitudes.



**Figure 8: Model generated utility of choices.**

Notions and expectations need to be significantly different from their normal values before an individual recognizes that incongruity as a “concern.” The level of discrepancy needs to evoke a response in line with “importance” assigned to information (notion). Because the perceived level of incongruity, intensified or diminished by attitude, contributes to the evaluation of choice, a perceived negative association may have a much larger impact on choice than an equal sized perceived benefit. This phenomenon is called loss or risk aversion and is exemplified in Prospect Theory or Risk Asymmetry (Kahneman and Tversky 1971, 1972, 1973a, 1973b, 1973c, 1974, 1979, 1979a 1979b, 1981, 1982, 1984, 1986, 1990, 1993a, 1993b, 1996, 1998, 1999, 2000, 2003).

The choice-making process in the model naturally produces the utility response noted in Figure 8. Figure 9 shows the theoretical and conceptual shape for the utility of choices in Prospect Theory.

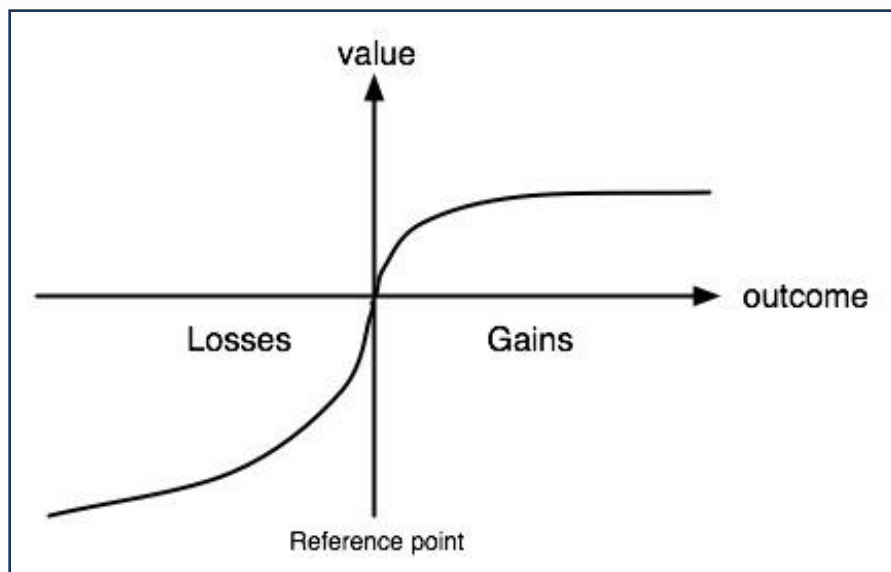


Figure 9: Conceptual-Theoretical view of Prospect Theory.

Because behavior is so varied, it must be produced by a complex assembly of rudimentary parts. If the parts were complex, they could not connect together, so apparently seamlessly, in so many ways. Metaphorically is it like a puzzle made of colored triangles that can fit together to produce any picture imaginable, versus a puzzle made of complex shapes that can only fit together one way. That the human brain is composed largely of only a few types of neurons gives credence to this perspective. Similarly, the psychological model is composed of a few basic structures, like Lego<sup>TM</sup> Interlocking building blocks, that allow a limitless display of complex organization and behaviors.

The basic equation that weighs information, be it simply sensory input or the multifaceted utility of alternative decisions, is shown below. It is based on Qualitative Choice Theory (QCT) whose foundation comes from psychology (Luce 1965) and from economics (McFadden 1984, 2000; Train 2003, Ben-Akiva & Lerman 1985). The first term (numerator) on the right hand side of the equation is the relative value of the utility (U) for a collection of information.<sup>1</sup> The second term (the denominator) is its comparison with all other relevant information. The result may be a choice (C), a simple recognition of sensory input, or an incongruity with a remembered condition and an existing condition. Subtleties of the equation reflect the probability that a choice is correct or useful in the context of perceived conditions.

<sup>1</sup> The "e" is the exponential function.

Choice Evaluation:

$$C(j) = e^{U(j)} / \sum_i e^{U(i)}$$

Equation 1.

The equation is also used for triggering learning and triggering behaviors by reflecting excitation and inhibitory responses. Conditions that are deemed too trivial to recognize or counter cause little excitatory reaction. Conversely, a condition may be so intense that sensory channels are saturated or that certain behaviors are too ineffective to execute. These situations cause extreme inhibitory effects.

When there is too much sensory input or too many activities needing behavioral responses, the same logic can prioritize choices to maximize the effectiveness of available cognitive processing (Anderson 2003).

The mathematics description of the utility function has a theoretical basis from the previously noted McFadden work and from Keeney and Raiffa (1976). The noted Train and Ben-Akiva work explains how to statistically estimate the parameters to the utility functions using historical data. Bradley shows how to use surveys and subject-matter-expert information to augment limited historical-data availability (Ben-Akiva and Bradley 1994, Hensher and Bradley 1993).

The next equation below determines the pattern strength of cues forming notions or the cognitive resources forming attitudes. It is directly derivable from the choice equation above when the utility is proportion to the logarithm of the cues, such as with the Weber–Fechner law (Weber 1978). As a specific manifestation of the choice equation, the notion (P), for example is a combination (z) of relevant cues (S). The  $\beta$  are the weights of each cue. and generally sum to unity.

Pattern Strength:

$$P = \alpha \times \prod_z S(z)^{\beta(z)}$$

Equation 2.

The next equation is an asymptotically exact, approximation of choice response when utility is based on a single consideration. It captures the incongruity between actual (perceived) conditions and expected (remembered or anticipated) conditions using an offset to avoid responding to insignificant discrepancies.

Incongruity:

$$D = \frac{Actual - Expected}{Expected} \pm Offset$$

Equation 3.

The last equation, below, is almost tautological in nature as the accumulation (R) of some quantity such as experience, memory, or capability that can atrophy over time ( $\tau$ ) in the absense of continued activity (Input). This equation is used in simulating notion assimilation, cognitive resource conditioning, and expectation formation. Its theoretical basis is found in the “stock and flow” constructs developed in the field of System Dynamics (Sterman 2000), but its statistical estimation and validation comes from the economics approach called Cointegration (Granger 1981, Granger & Engle 1987).

Conditioning and Fading activity:

$$R(t) = \int_{t_0}^t (Input(t) - R(t) / \tau) \times dt$$

Equation 4.

Causal chaining and branching of the processes described with these equations produce all essential psychological phenomena noted in Figure 7.

The section below further describes the key components of the psychological model as portrayed in Figures 6 and 7.

## 5. Modeled Elements of Behavior

The next several sections identify the elements contained in the simulation model. A detailed description of each element, including equations, is provided in the appendix referenced for each section

### 5.1 Attitudes

Attitudes are pattern derived from cognitive resources. Attitudes can be affective or rational. Some types of attitudes have evolution-based and cultural based components. In essence, attitudes are an internalized notion affecting the interpretation of external cues. Figure 10 display an illustrative example (using only two cognitive resources) showing how increased cognitive resources (such as a reinforced dislike for residing American troops and the recognition of diminished living standards) leads to an attitude of blaming the Americans for the existing condition (despite the destruction being caused by fellow countrymen). Expectancy Value Theory (Fishbein 1961, 1962, 1963, 1975) argues that attitudes are the basis of beliefs and values. We use the more generalized weighted product approach to modeling attitude rather than simple sum used in the original research. The change in the intensity of the attitude with a change in cognitive resources depends on the relative weight (the  $\beta$  in the Pattern Strength equation) of each cognitive resource to the attitude (Bosse 2010). Other cognitive resources can modify the relative weight over time, such as cognitive resources related to religious attitudes or to the acceptance of other sects within the country. In this example, the attitude increase moderately with increases in both cognitive resources. See Appendix 4 of the equations describing Attitudes.

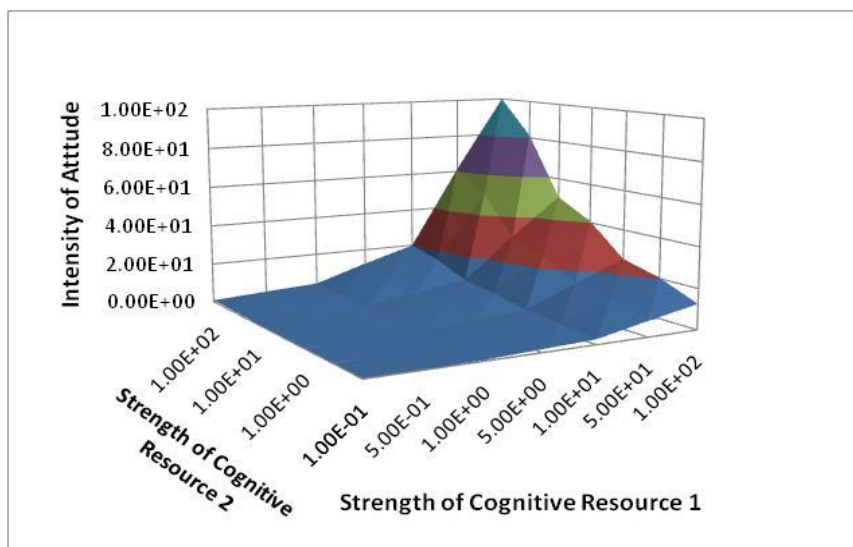


Figure 10: Attitude Intensity

## 5.2 Notion Formation

A collection of Cues forms a Notion. Notions are then a collection of selected stimuli relevant to specific intents and behavior. Notion can be affective or reasoned and influence the utility an intent (choice) or behavior has. The sensory aspect of notion formation is identical in construction to attitudes but is composed of the cuing stimuli rather than cognitive resources. Nonetheless, attitudes (by altering the  $\beta$  of the pattern strength equation) can affect how cues produce a specific notion. In an identical fashion to Figure 10, Figure 11 provides an illustrative examples of how a collection of cues (in this instance two), produce of notion of specific intensity.

Elaboration Likelihood Theory (Petty & Wegener 1981, Petty & Cacioppo 1986, 1999) considers notions as based on differing patterns of perception that then feed into the utility of actions whose components may include higher cognitive considerations as well as emotive elements for support or biasing of intentions. We apply the theory in exactly this way within the computational model. See Appendix 7 for the detailed mathematics of Notion Formation.

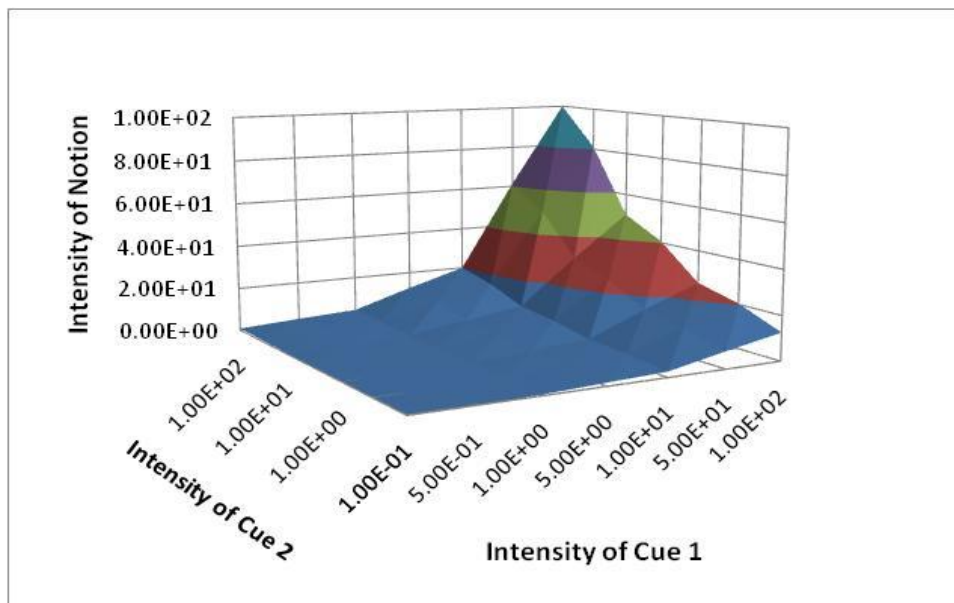


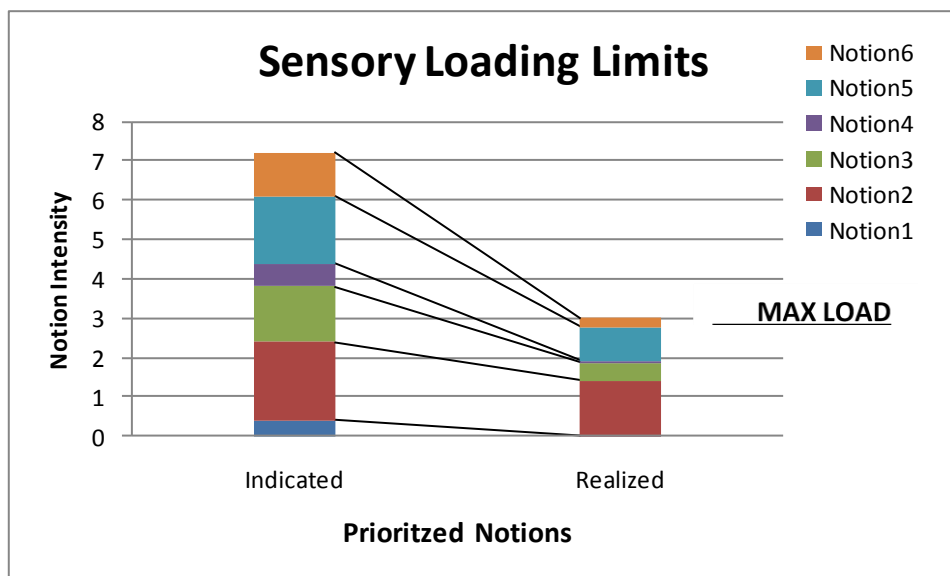
Figure 11: Notion Intensity

## 5.3 Notion Prioritization

Limited attentiveness capacity requires a prioritization of notions to avoid being overwhelmed (incapacitated). The choice equation naturally simulates the prioritization sensory information



(Cacioppo 1999, Izard 2009, Greenberg 2003). As in the previous sections, historical data and information from subject matter experts can directly calibrate the equations. Figure 12 displays how sensory input produce indicated notions that total to an intensity above the maximum cognitive loading. The prioritization process reduces the total loading to the maximum acceptable level. See Appendix 7 for the mathematics defining Notion Prioritization.



**Figure 12: Notion Prioritization**

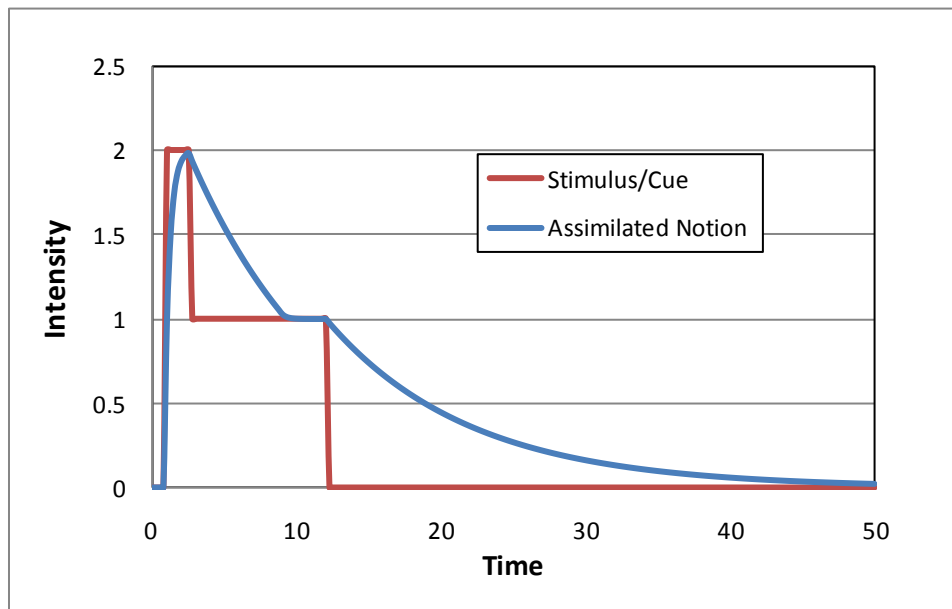
## 5.4 Notion Assimilation

Psychological studies and experiments indicate that individuals only remember the peak and end value of sensory input (Fredrickson 2000, Schwarz 2000). Further, the peak values from multiple sensory episodes are not additive. Other studies indicate that cuing frequency and recency play a role in behavioral responses (Perugini 2001). As noted previously, the lingering aspect of sensory notions (certainly associated with recency) can have an emotive content called "mood." The speed at which a cue becomes a notion is faster for an emotive notion than a reasoned notion. The lingering effect is longer for an emotive notion than reasoned notion. Hence, there can be mood-congruent behavior where the phasing of reasoned and affective notions play off each other.

The cascaded use of the conditioning equation noted earlier produces exactly these psychological responses (Busemeyer 1993, Grossberg 1987). Figure 13 shows a varying set of sensory notions and the resulting assimilated notion. The blue line denoting the assimilated notion shows a fast rise in this instance that is solely dependent on the end point of the sensory input. The notion lingers and a second set of sensory input (the second red horizontal line) is not additive with the stimulus. It does have its final value as the starting point of the lingering emotive notion, no matter

how long the stimulus lasts, consistent with the psychological theory and experimental results (Forgas 1999, Gratch 2004).

See Appendix 7 for the mathematical description of Notion Assimilation.

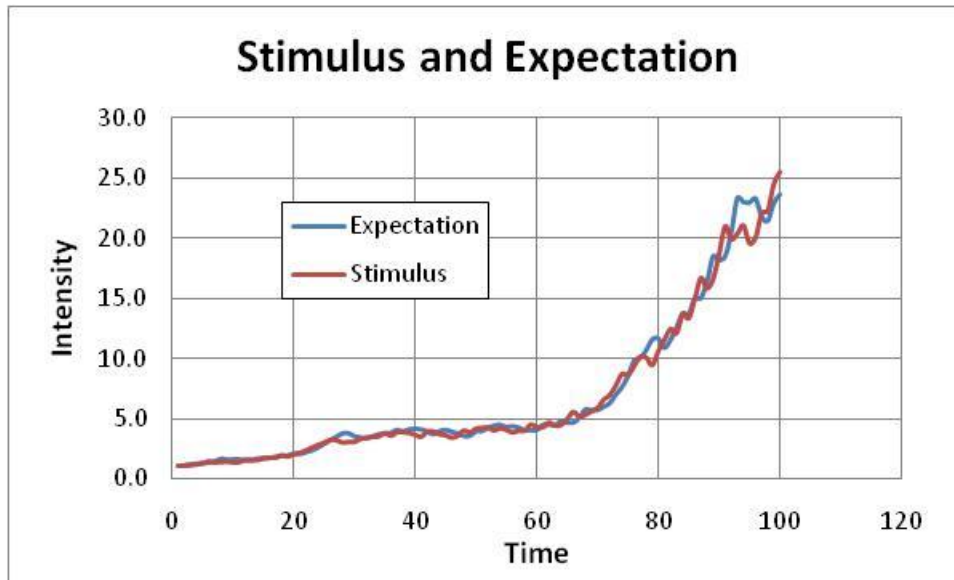


**Figure 13: Assimilated Notion Dynamics**

## 5.5 Expectations

Incongruity comes from the difference between perceived conditions and expected conditions. Expectations come from the memory of prior conditions. Nonetheless, the expectation for the future may not coincide with the exact memory of the past conditions if there is anticipation of change, such as raise or a job promotion. Further, the formation of expectation has to smooth out the noise from routine variations, such as in summer temperatures or the value of the stock market. The parallel use of multiple conditioning equations produces all these phenomena. Figure 14 shows the simple instance where the expectation is simply an averaging of historical values in a very noisy environment. The expectation formation process acts to smooth out fluctuations so that the use of expectations to determine incongruity is a legitimate expression of the disparity between current conditions and "normal" conditions.

The mathematical description of Expectations is presented in Appendix 8.

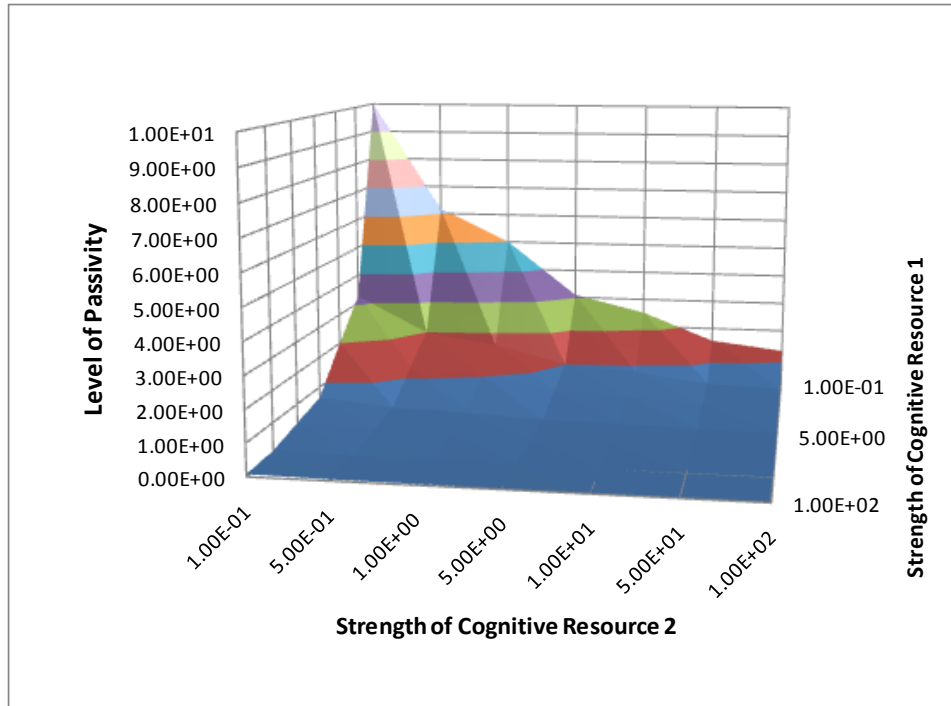


**Figure 14: Expectation Formation**

## 5.6 Passivity

Passivity is simply an attitude that affects the offset associated with incongruity. In a sense, passivity is an attitude toward incongruity. A high degree of passivity means that there needs to be a large disparity between existing and "normal" conditions before the individual recognizes a need to act. Passivity determines the changing sensitivity to incongruity. Estimating the parameters of passivity would require an extended data time-series, but the impact of passivity is secondary and can be neglected for most studies. Figure 15 shows how cognitive resources affect passivity. In this instance, the depicted response is a mirror image of attitude formation with a lower set of cognitive resources causing a much greater level of passivity. This situation can be interpreted as an individual not yet "tuned" to recognize the importance of certain notions.

Appendix 5 provides the mathematical representation of Passivity.

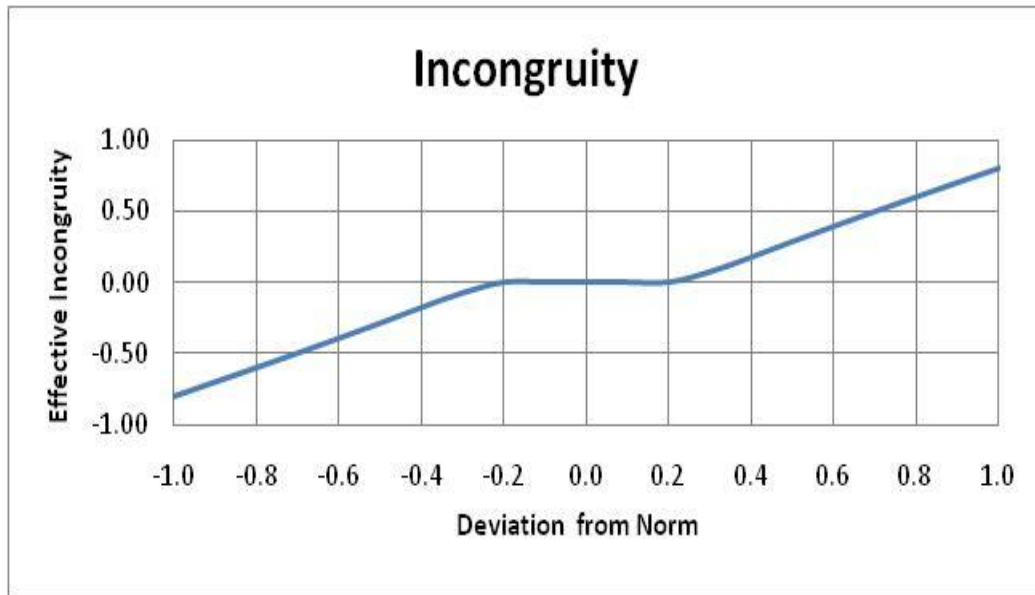


**Figure 15: Passivity intensity**

## 5.7 Incongruity

Incongruity is the proportional change between perceived conditions and expected conditions. It is the primal dissonance driving the reaction to external stimuli (cues). It has a dead-zone response using an offset (as discussed above) to capture the threshold effect. Figure 16 shows how incongruity changes as the proportional difference between actual conditions and expectations vary. A 20% threshold (offset) is used in this example. The incongruity between existing conditions and expectations motivate behaviors to reduce the incongruity (Festinger 1956, 1957).

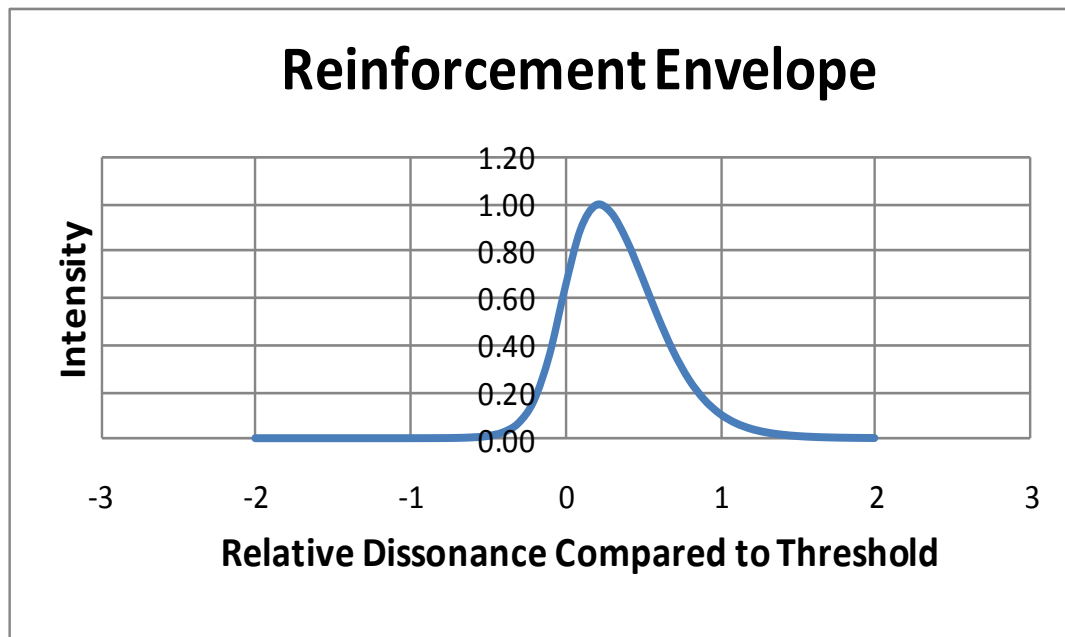
Appendix 5 provides the mathematical representation of Offsets, while Appendix 6 presents Incongruity.



**Figure 16: Incongruity Realization**

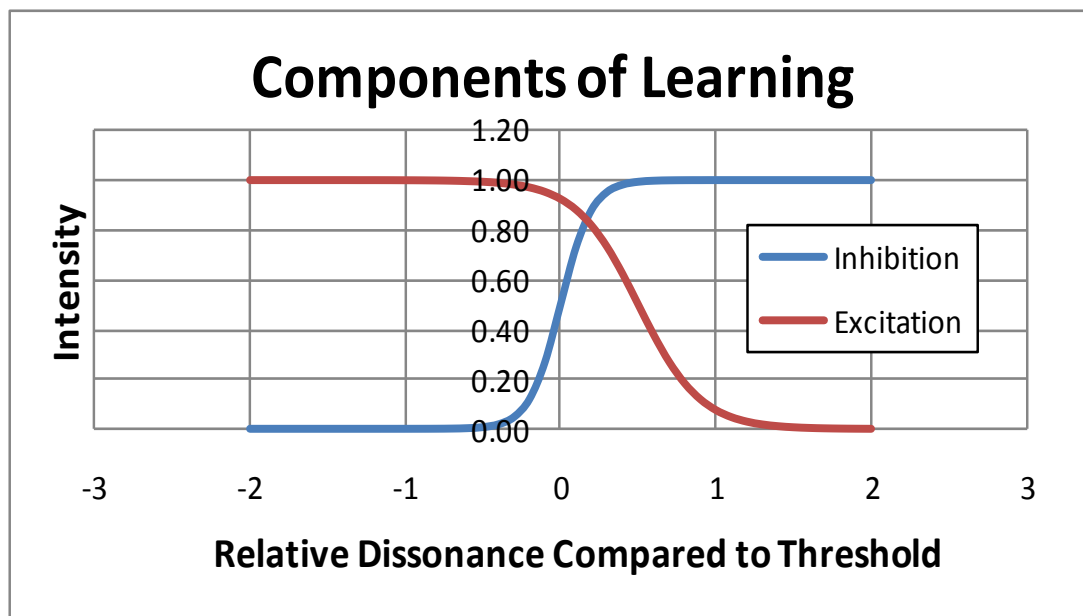
## 5.8 Cognitive Resources

Cognitive resources are the accumulation of experiential learning tempered by evolutionary constraints. They can contain emotive conditioning (such the fear of dark alleys) or the acquisition of a physical capability (such as playing a musical instrument). Incongruity initiates learning. Learning changes the process for coping with the environment. If the incongruity is small, it means that individual is well suited to respond to existing conditions and has probably controlled the existing condition to correspond to expectations -- through previous behaviors. For example, a worker has a perceived adequacy of income based on the "working" behavior to generate income. If the incongruity is too great, there is little incentive to learn because no immediately realizable amount of learning (augmented cognitive resources) can overcome the existing incongruity. For example, there is little to do if a large asteroid suddenly heads your way out of the sky. Thus, there is a window of incongruity that contains adequate motivation for conditioning the cognitive resource. This "window" is depicted in Figure 17. The maximum learning occurs when the incongruity is significant but also one where additional cognitive resources could generate behaviors to bring physical conditions (notions) back within the range of expectation through a modest amount of learning. The "motivation" can come from both emotive and non-emotive incongruity (Phelps 2004).



**Figure 17: Conditioning Window**

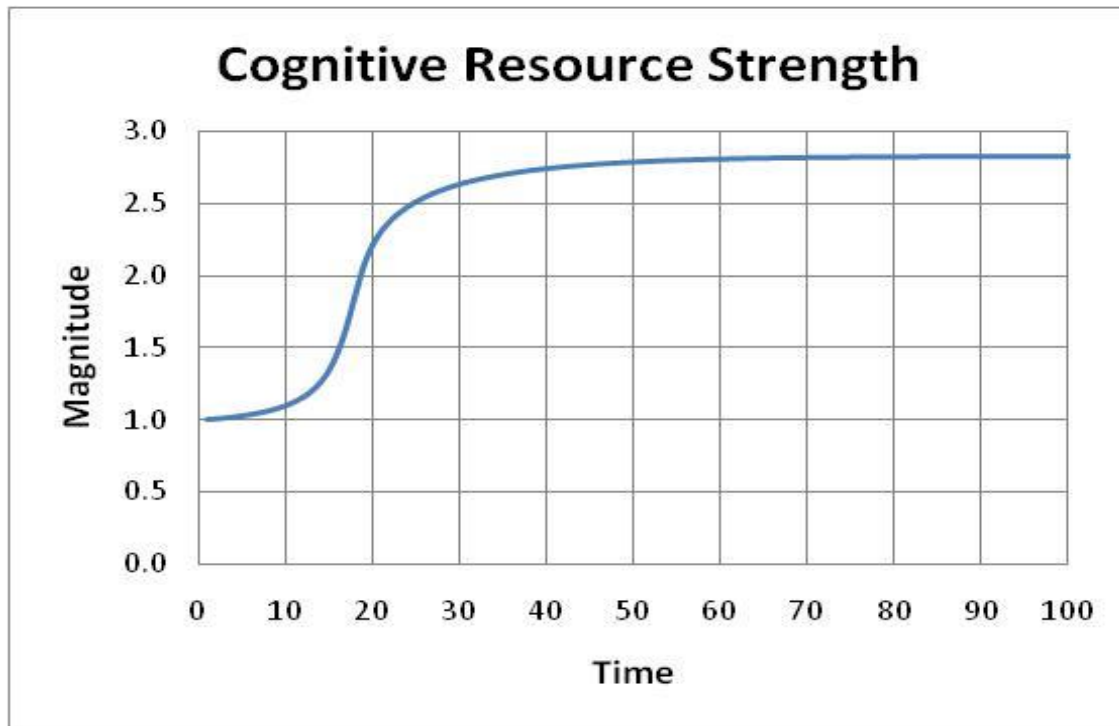
Figure 18 shows the inhibitory and excitatory components that produce the conditioning window. These components are simply a result of two additive choice equations.



**Figure 18: Components of the Conditioning Window**

Conditioning improves the level of cognitive resource through the conditioning response depicted in Figure 19. The conditioning can improve a cognitive resource until behaviors bring incongruity levels to within acceptable ranges. The level of a cognitive resource grows to slightly exceed the level needed to accommodate external stimuli. This phenomena has a basis in evolution where

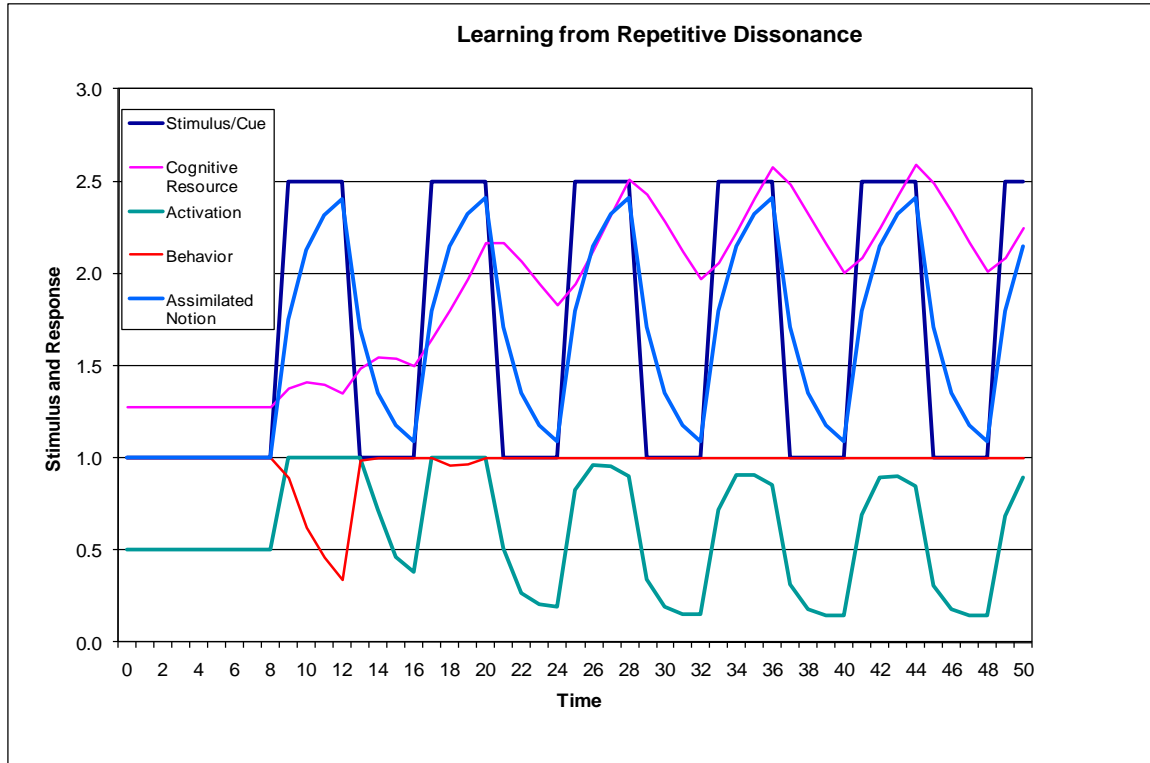
there needs to be a contingency if allows the individual to tolerate conditions that exceed previously experienced values. In brief, repetitive tolerable, stress producing (incongruity) events modify cognitive resources to cope with that environment.



**Figure 19: Growth in a Cognitive Resource**

Figure 20 shows the simulated cognitive resource response of a new accountant as she suffers through quarterly budget cycles (the dark blue "stimulus" line). Initially, the perceived demands plus "assimilated notion" line) are nearly overwhelming with a decline in effective behavior (the red line). But in subsequent episodes there is continued learning and improved performance (the magenta line). Eventually, the incongruity causing activation (the turquoise line) has a declining trend (Backus and Glass 2006). The cognitive resource itself would have a response similar to that shown in Figure 19.

The equation capturing the dynamics of conditioning and cognitive resources is provided in Appendix 9.



**Figure 20: An example of Capability Learning.**

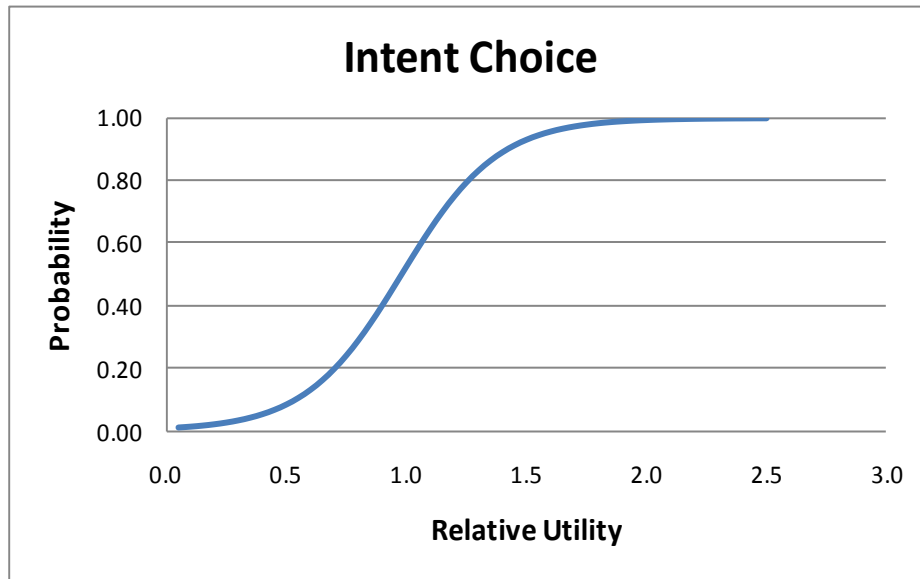
## 5.9 Evaluation and Selection

Choice is based on perceived utility of action. If the choice process contains two options, and one option (#1) has a constant utility, while the second option (#2) has a increasing utility over time, Figure 21 shows how the probability of selecting the second option as its utility changes relative to option 1. This logic works with any number of choices and with any level of complexity for the utility. The mathematical construct was developed within the economics community but is consistent with psychology (for example see the work of Cacioppo & Gardner 1999, Mellers and Schwartz 1999, Busemeyer & Diederich 2001, Loewenstein 2001, Mellers 1999, Smith 1985). The mathematical model is notably robust to limited data used in its parameterization.

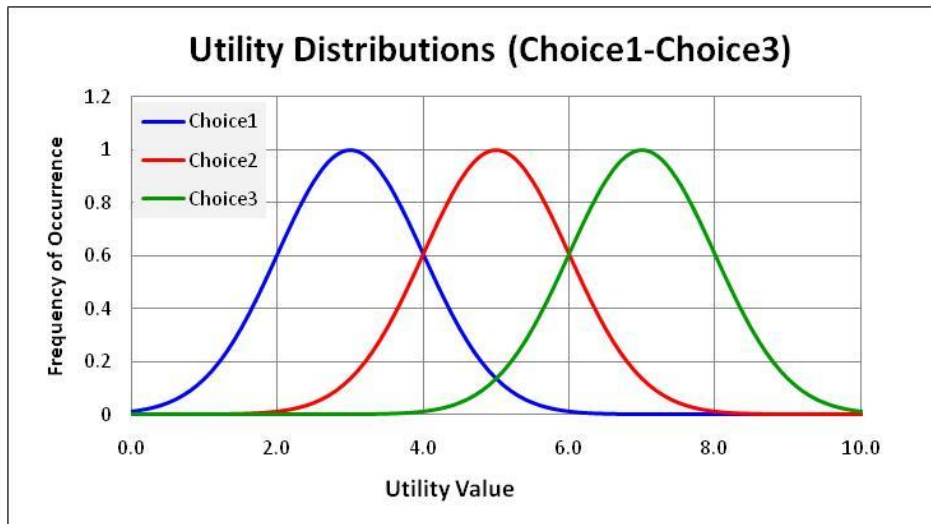
Figure 21 illustrates the utility distribution for three choice options. In this example, the choice is to select one of three technologies (such a car brands). At any instant, the utility is actually an uncertain quantity. The individual selects a particular item with a probability equal to the chance that one item is perceived as having a greater utility than the other items. In the Figure 21 example, utility is assumed to be inversely proportional to price. Therefore the individual has a high probability of selecting the technology represented by the blue distribution (technology #1). However, there is a significant probability of selecting the other two technologies where the distributions overlap and the individual may perceive a higher cost technology as less expensive than technology #1.



Choices are often made with limited information that generally has emotive as well as reasoned content. Bounded rationality limits the amount and use of information (Simon 1957). The information is imperfectly transferred to different entities and this affects the decision process of the individual entity (Stiglitz 1985, 1986, 2002). The qualitative choice process used in the model readily simulates these aspects of decision making (see McFadden, Train and Ben-Akiva references). Appendix 10 presents the detailed mathematical formulation of the evaluation and selection process.



**Figure 21: Choice evaluation:**



**Figure 22: Uncertainty in the utility of different choices.**

## 5.10 Indicated Behavior

The evaluation process decides what choice to select, that is, it decides the intent. It does not resolve whether the intent will be executed, nor does it establish the intensity of an executed behavior. Using the same logic as discussed for cognitive resources, there is a window of incongruity that governs the triggering of behaviors, as shown in Figure 23. Similarly, Figure 23 is the result of additive inhibitory and excitation processes as shown in Figure 24, and comparable to the process shown in Figure 18.

The complete mathematics of Behavior simulation appear in Appendix 11.

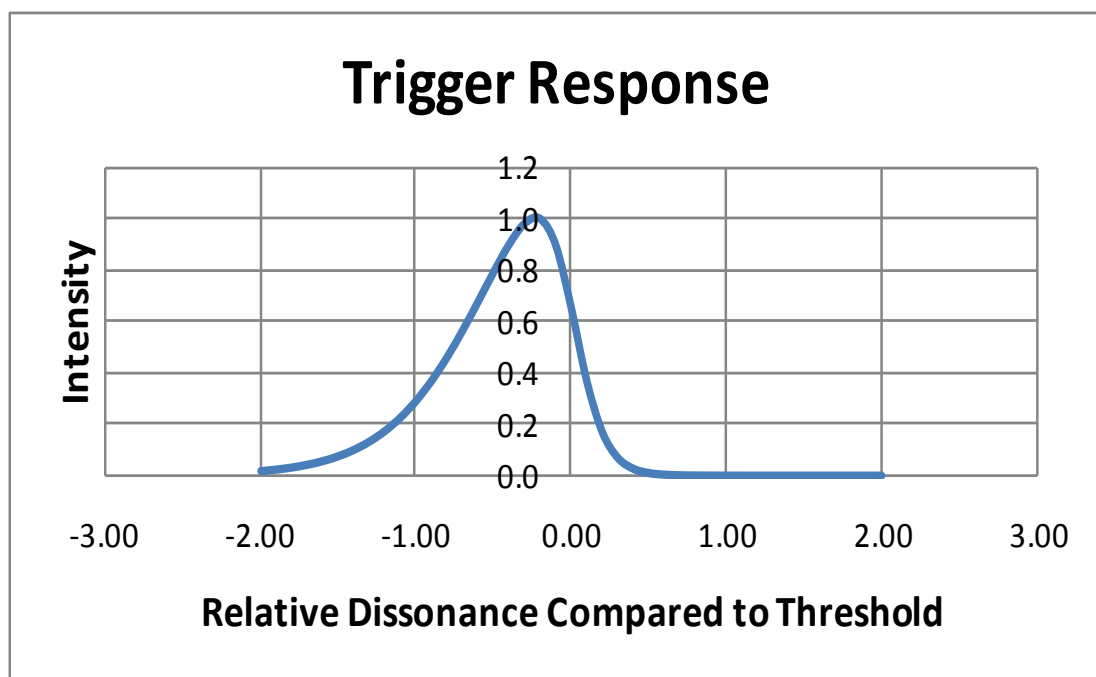
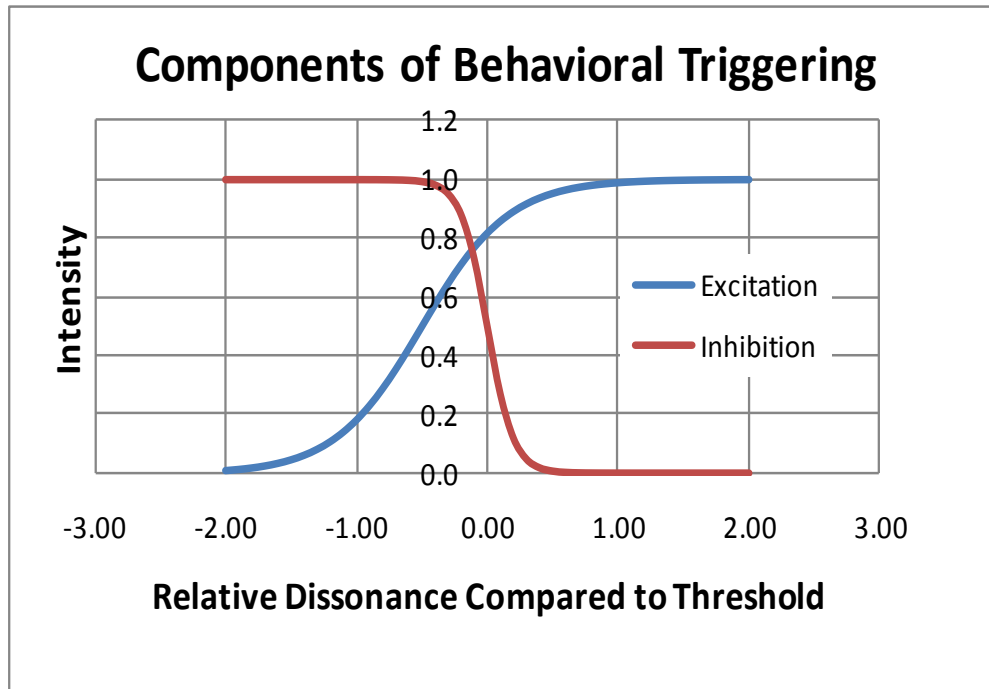
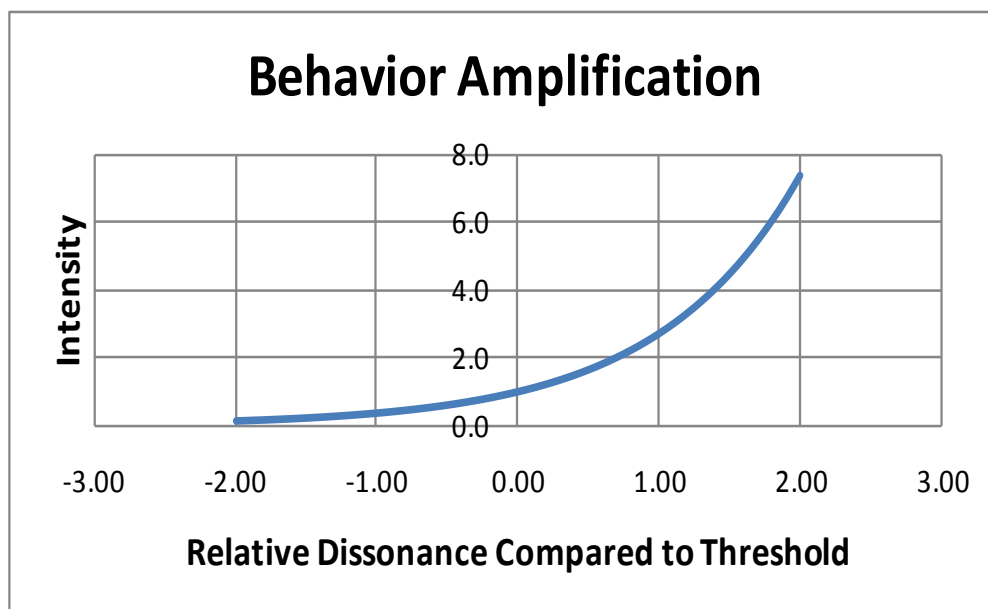


Figure 23: Triggering Window



**Figure 24: Components of the Triggering Window**

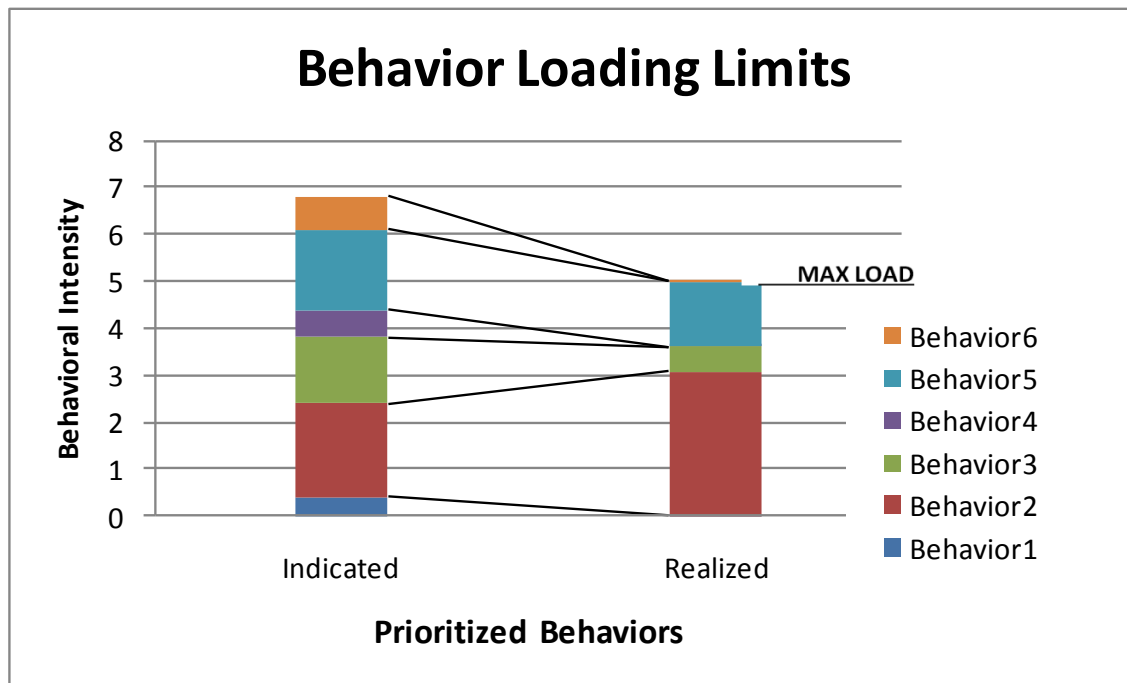
While the values shown in Figure 23 determine the fundamental triggering of the behavior, emotion-based notions and incongruity can amplify the intensity of the resulting behavior (Isen 1998, Zeelenberg 2002, Anderson 2003, Marsella 2010). This amplification process is illustrated in Figure 25. Note that the amplification is just the initially rising response of the choice equation as can be recognized in the left hand portion of Figure 21.



**Figure 25: Behavioral Intensity**

## 5.11 Realized Behavior

Limited response (energy) capacity requires a prioritization of behavior to avoid unproductive (ineffective) responses. This phenomenon is identical to that for notion prioritization (Figure 12), except that it prioritizes behaviors. Figure 26 shows how the process can limit and select the key behaviors to stay within the limits of an individual's capability to respond.



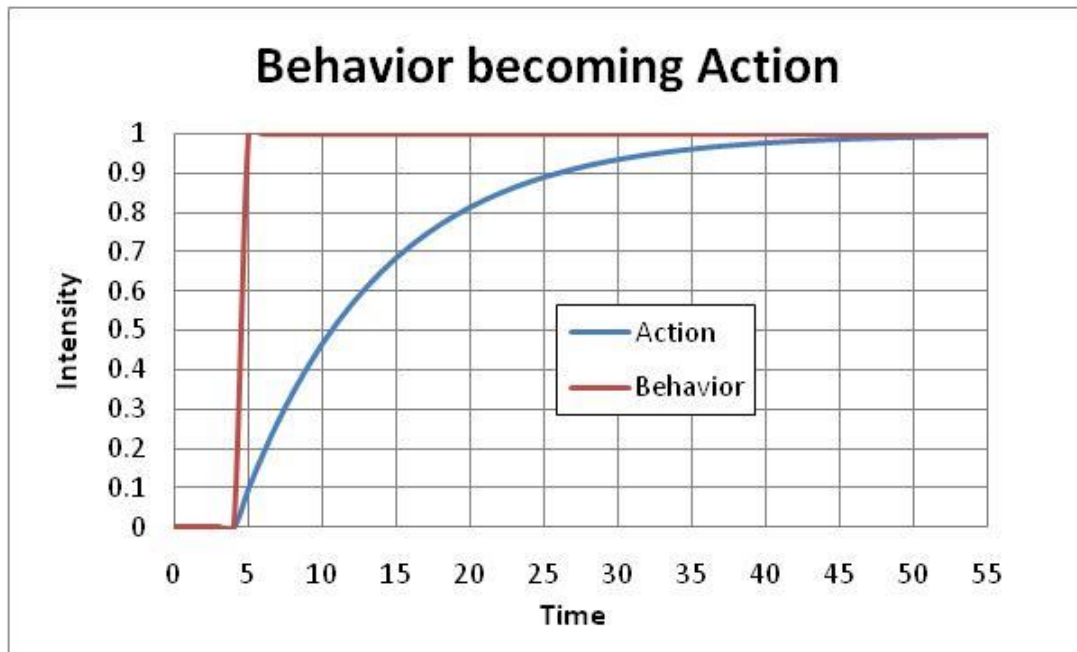
**Figure 26: Behavior Prioritization**

Appendix 11 contains the mathematics describing Behavioral Prioritization

## 5.12 Action

It takes time for a behavior to have an impact on the physical world where other individuals and the physical world itself can realize and respond to the behavior. Figure 27 shows the probabilistic delay between the time a behavior is initiated (red line) and the time it is fully realized as an action (blue line). This dynamic is just the result of the conditioning equation discussed previously.

The mathematics defining the Action process are in Appendix 12.



**Figure 27: The Delay between Behavior and Realized Action**

### 5.13 External Conditions

The psychological model described above can connect directly to engineering or economic models that simulate physical world conditions. The actions of individuals represent activities (such as construction work) that directly affect physical conditions. The physical conditions in turn represent stimuli (such as the employment opportunity of a newly opened factory) to other individuals. The psychological framework contains a use-specific model of the physical world for completeness that is calibrated to simulate the actual physical processes. However, independently developed physical models can be linked to the psychological framework.

For the casual logic of the model to work consistently, the external conditions simulation needs to be structural, with the simulation moving through time and only using current or historical data. This approach also implies a structure compatible with a system of algebraic-integral equations. This further implies the physical (external) model is initialized for historical or current conditions. Despite the fact it is discussed last, the physical model is the first procedure of the executed in each time step of a simulation.

An example of modeling External Conditions is presented in Appendix 14

## 5.14 Signal Dissemination

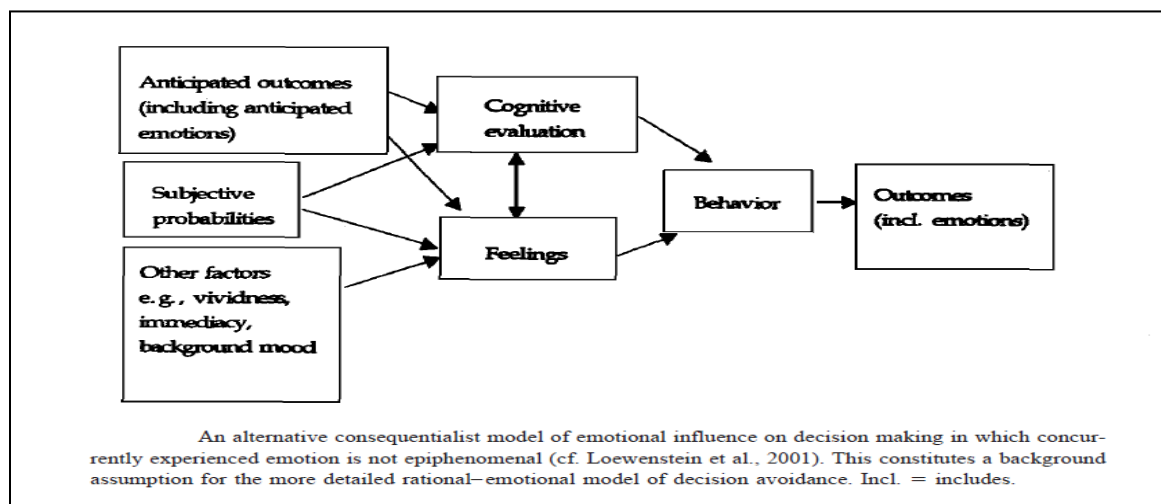
Actions and External Conditions are stimuli (that become cues) that start the process over again through time. Signal Dissemination is simply a representation of the social network that connects information from one entity or physical condition to another entity or physical condition. It disseminates the information. Not all entities are aware of some information and not physical conditions affect everyone. The behavior or results of behaviors can again act as stimuli to the originating individual as part of the feedback process causing the learning to change the external or internal (learned cognitive) conditions to reduce dissonance (Bandura 1977). Further, associated individuals (defined via the social network), may see an action as part of shared successful outcome which further promotes “herd” behaviors that self reinforce societal and physical conditions (Rotter, 1945).

The mathematics describing Signal Dissemination are provided in Appendix 13.

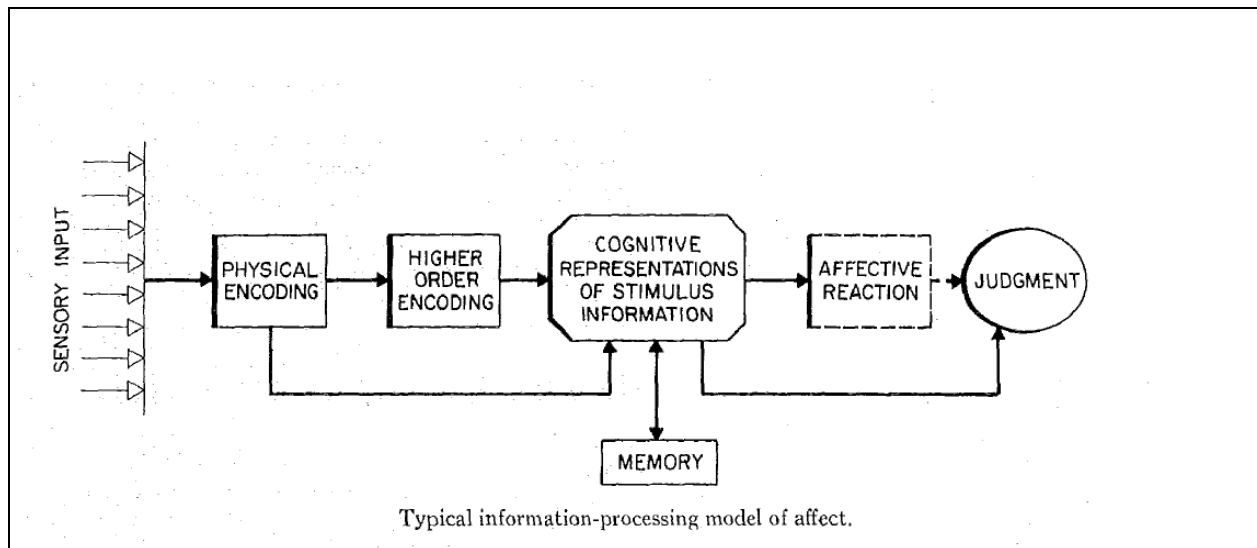
## 6. Psychological Behavioral Economic Compatibility

Figures 6 and 7 showed the overall components of the psychological model while the preceding sections further described each of the components. Figure 27 shows the psychological understanding of emotion and reasoned interaction to produce behavior (Smith 1985, Frijda 1988, Loewenstein 2001). To a large extent, such theoretical psychological conceptualizations do not result in computation models that allow the testing of hypotheses or the realistic prediction of behavior. Computational models that do exist generally have simple algebraic representations that do not attempt the time dependent nature of the behavioral phenomena. The psychological model described here reliably captures the psychological theory and experimental results implied in Figure 27.

Figure 28 presents another theoretical view of the decision-making processing that additionally includes memory as a changing component of the process (Zajonc 1980). The psychological model described in this report implements the conceptual model in its entirety. These examples of Figures 27 and 28 provide further evidence that the theories used to develop the computation model represent a unified capability to realistically simulate behavior consistent with and calibrated to the available data characterizing individuals and groups of interest.

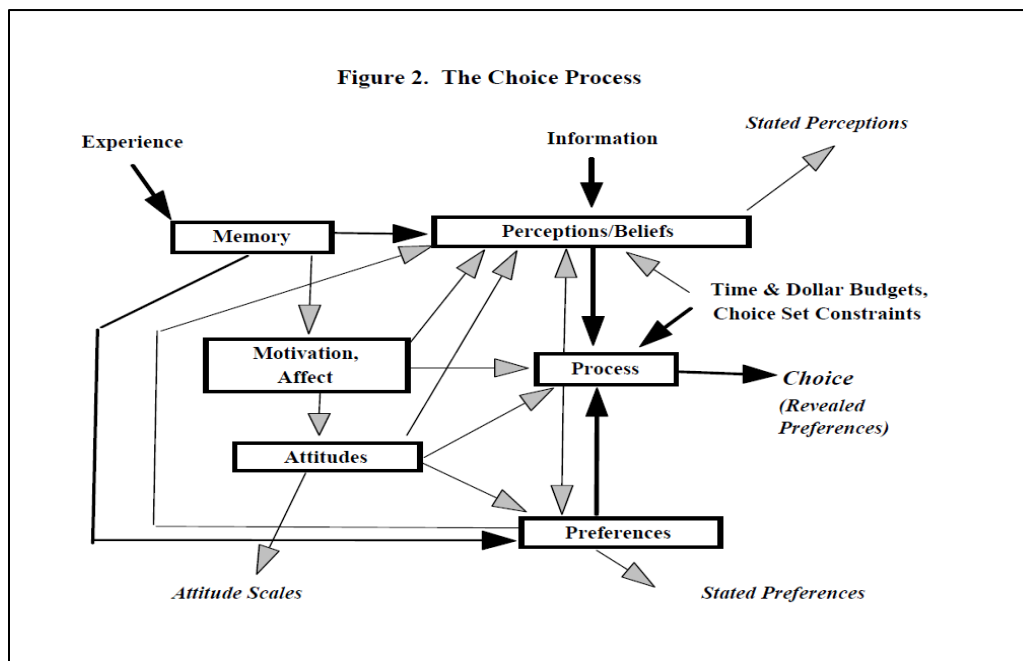


**Figure 27: Reason and Emotion in Decision-making (Loewenstein, 2001)**



**Figure 28: Information processing and cognitive resources in decision-making (Zajonc, 1980)**

Similarly, conceptual efforts within the economic community to comprehensively describe human behavior have not been extended to comprehensive computational models. Figure 29 shows an enduring and well known conceptualization (McFadden 2001). Note its comparability and compatibility with Figure 6 and Figure 7 depicting the SNL psychological model.



**Figure 29: Economic Choices as a Psychological Process (McFadden, 2001)**



## 7. Individual vs. Group Representations

The ability to apply the psychological engine across the domains of individuals and groups comes from the characteristics of the QCT approach that defines the choice process. Choice-making is at the heart of the psychological engine. The use of QCT for determining the choices is well-founded (McFadden 1984, 2000; Train 2003, Ben-Akiva & Lerman 1985). In the model, we have routinely changed what would normally be a constant parameter into a term that linearly changes (amplifies how notions determine the choice) with the magnitude of cognitive resource (the norm, belief, or emotive content of the decision). We let any non-linearities be in the reinforcement (conditioning) of the cognitive resources where theoretical bases for such reinforcement exist. The QCT Multinomial Logit and Probit approaches are consistent with the Tanh-1 logic of neural nets and its use in other SNL cognitive models (Manski 1985).

We also use QCT to determine the fraction of any cognitive realization that flows to the next component along the stimulus-response paths. If the “output” of any cognitive realization (such as a notion) is zero, then in practice, it does not matter what fraction of “no response” is transmitted. But in the absence of other notions or stimuli, it may be realistic to imagine that the mind would inhibit any aspect of response realization. Further, in the absence of information, the mathematical formulation of QCT produces a probability of response that is equal among all possibilities. That is, if the choice is to do or not do, the probability goes to 0.5, whereas no stimuli maybe should indicate either a 100% response or no response. We use an amplifier term to ensure the complete response to no input.

The QCT utility formulations are currently quasi-linear (e.g. an ordinal-utility term might be “ $\alpha \cdot S$ ”) but there is a strict assumption that input data is ordinal and proportional. That is a stimulus (S) with a value of 6 has 3 times the importance of a stimuli of 2. The stimuli are relative quantities with the intent to use them exclusively with affine mathematics. Affine mathematics as used here indicates the results of the model are independent of affine transformations or arbitrary numeraire-based scaling. Because there are no absolute referents in cognitive science, all quantities can only be stated in proportional relationships to other quantities. This practice is difficult to implement consistently. The use of logarithmic terms guaranties the sense of proportionality and affine mathematics (such as “ $\alpha \cdot \ln(S)$ ”), but creates numeric (not QCT theoretical) problems when S goes to zero. A solution is to argue that “mental” noise always maintains a background at minimal discernable sensory levels. Such an approach removes the limiting assumptions of QCT and optically presents a logic consistent with the QCT, psychological, and economic (value-based choice) foundations of the model. With this approach, all independent variables need only have a lower (theoretical) bound of zero.

We use the raw data to parameterize the equations. As discussed In Appendix 16, dynamic and static data should exist for estimating and calibrating static and dynamic aspects of individual

components of the model shown in Figure 7. Most often, there will not be data to estimate the entire dynamic system. Static estimation of the entire system's parameters will generally be possible. Estimation is the process to develop fixed parameters. Because the model contains feedback, all parts must be exactly self consistent. Calibration exactly matches the model to specified output data, given the specified input data. Calibration typically requires an inverse model, but the forward model can be used through an iteration process – if the parameterization iterations are convergent.

Specifically, the use of the linear utility function does not imply a priori correctness. As data dictates, the use of the logarithmic or linear function needs routine reevaluation. For the logarithmic function, a unity value of the independent variable is the neutral value; for the linear function it is 0.0. Large values of independent variables within the logarithmic function provide diminishing returns on the weight of the decision; the linear function increases the utility linearly. An asymptotically 0.0 value for the independent variable in a logarithmic function, drives the exponential of the utility to 0.0; the linear function requires the independent variable to asymptotically go to negative infinity for a zero (exponential weight) of the utility. The choice of linear or logarithmic representations primarily affects the probability of a choice under conditions far from average values. The data used to derive the function form and its parameters are equally obtainable from time-series, panel, individual or group data.

QCT, by design, is thus equally applicable to individuals and groups. At the individual level, the selection process is a probability of a choice; at the group level it is the fraction of the entities within the group making the choice.

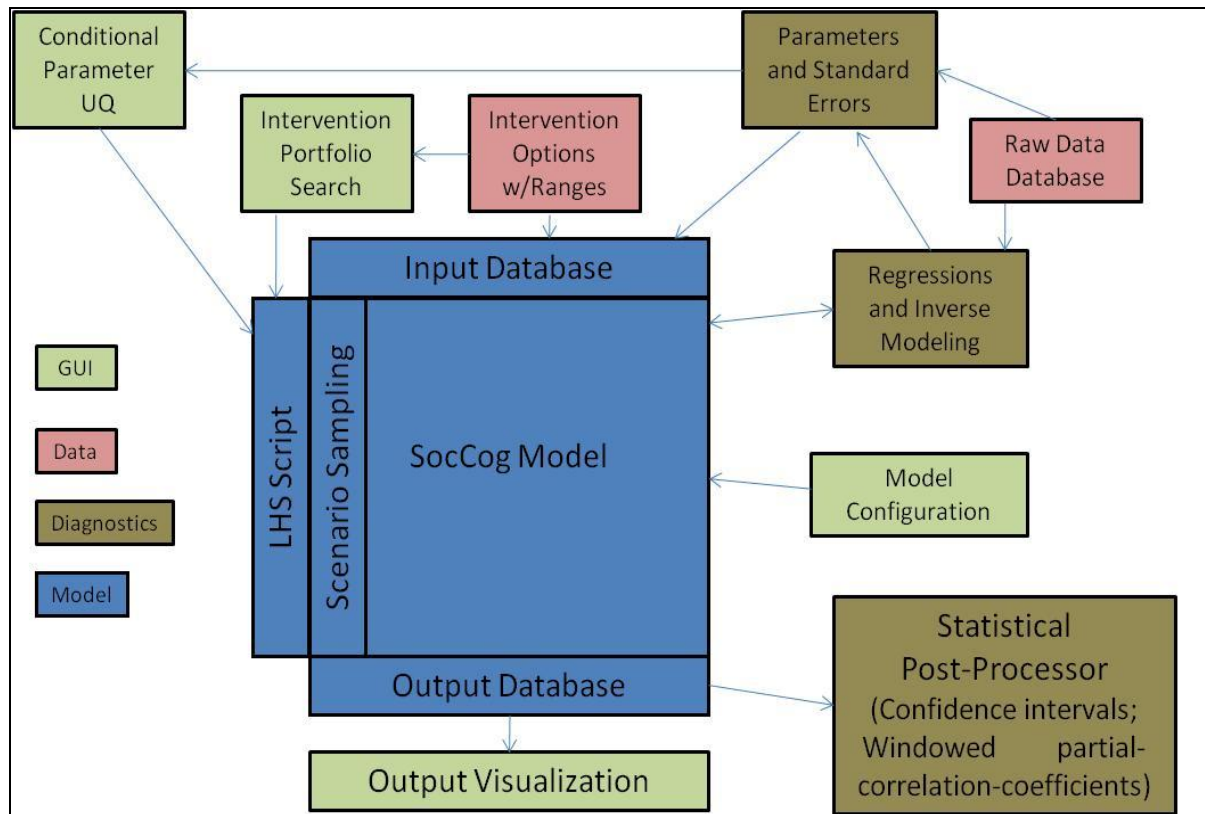
## 8. Using the Psychological Engine for Analysis

The psychological model is contained within a larger analytical framework as depicted in Figure 30. Historical data and SME information become the raw data used to calibrate and parameterize the model. The uncertainty in the data is explicitly determined through the statistical process used to develop model parameters. Appendix 16 provides information on parameter estimation.

With this statistical knowledge, we can provide confidence intervals on the results of the model analyses that test interventions. By simultaneously performing uncertainty quantification for model parameters and potential interventions, the framework can determine the portfolio of interventions that have the highest (quantified) probability of success despite uncertainty. It can also quantify the risk associated with the intervention not performing as anticipated. Additionally, as will be discussed shortly, the framework can perform sensitivity analyses to determine what minimal additional information is needed to maximally reduce uncertainty and further assure the proposed interventions produce the desired outcome throughout the time horizon of interest.

Because the model is causal, decision-makers can reach-back into detailed results of the simulation to independently evaluate the nuanced processes that caused the predicted outcomes. Moreover, the same process can determine early warning fingerprints whose measurement today or during the initial implementation of an intervention can verify or exclude the possibility of critical conditions/outcomes.

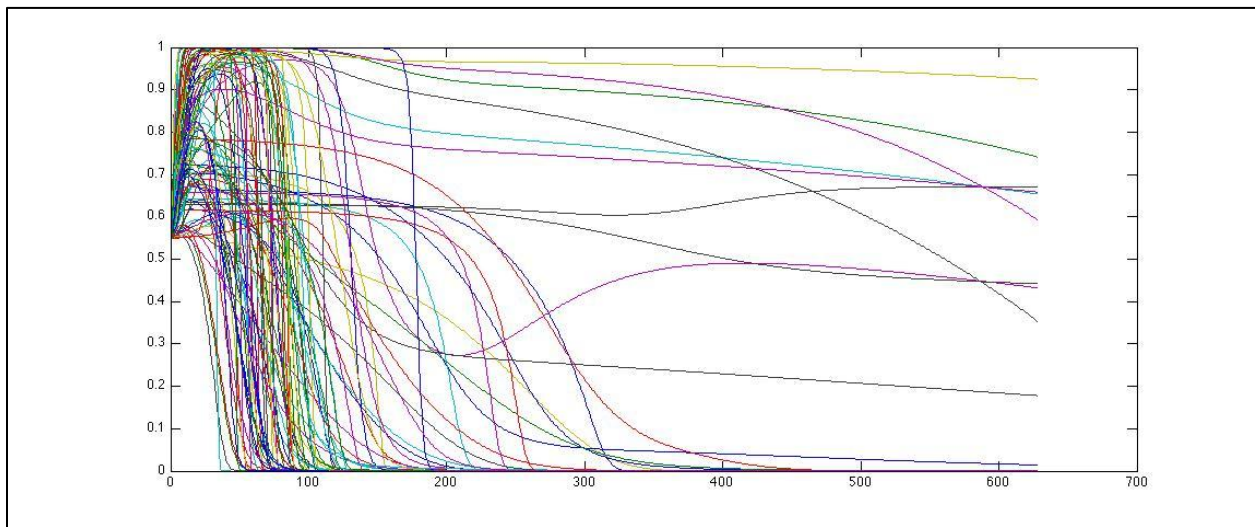
Appendix 3 supplies a more formal and computer-science oriented explanation of the model organization.



**Figure 30: Analysis Framework**

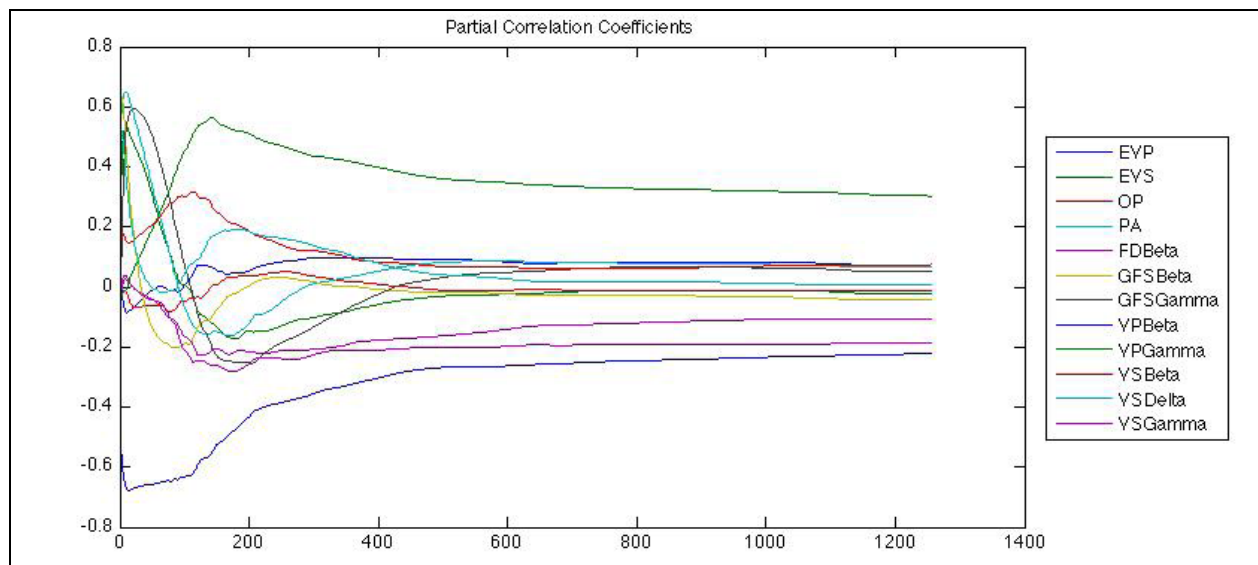
## 9. Intervention Confidence

Figure 31 shows the uncertainty quantification of changes in voter support due to changes in a foreign government's policies to subsidize food costs. A complementary diagram showing protest activities (not included here), demonstrates how an integrated analysis almost invariably reveals the non-sustainability of such a policy. The collapse of voter support occurs over most of the probability space. There are however a small number of outlier situations where the collapse does not occur. A causal reach-back of the analysis indicates under what (few) circumstances the desired outcome does not occur. Secondary interventions can prevent these circumstances from occurring, or conversely, the verification of such existing circumstances would indicate that an alternative intervention strategy is needed. This process can greatly limit blind-siding and ineffective interventions.



**Figure 31: Confidence analysis.**

Figure 32 show the sensitivity of a desired outcome (in this instance loss of voter support for a "dictator") as a function of 1) intervention points, 2) uncertain data on leader and public behavior, and 3) other conditions within the society over time, as the intervention impact unfolds. With such information, operations can adapt their intervention as needed to maintain control and the desired momentum towards a goal. The existence of both negative and positive leverage points (as shown in Figure 32) allows a large degree of control. Knowing that these leverage points change magnitude and direction, over time as a function of changing ground conditions, further manages the confidence a decision-maker can have in executing interventions.



**Figure 32: Adaptive Control of Interventions**

The model can be used to show how to unobtrusively "ping" the real world to obtain the data needed to refine parameters for increases confidence (less uncertainty) in the recommendations or results from the model.

## **10. Summary**

This document has provided a brief discussion of the SNL psychological model. It has shown that the model provides a data-driven analytical capability, consistent with psychological and economic theory, usable for assessing kinetic and non-kinetic national security operations. We are still completing early testing of the framework and developing the secondary components needed for usability. Most of the sub-components have been thoroughly tested in previous studies. Still, the integration of all the parts into a comprehensive system is a research effort. Within the next few months, we will be testing the model with detailed data sets and realistic intervention scenarios. Our, albeit limited, experience with use of the model to-date indicates that the approach we are taking is sound and can produce the expected capabilities.





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## Appendix 1: Glossary

[Terms denoted in **bold** are also defined in the glossary.]

*Action:* The physical realization of a **behavior** in terms of the external **signal** it provides to the physical environments and other **entities**.

*Amplification:* The intensification of a response due to emotive/cognitive **attitudes, incongruity, or notions**.

*Assimilation:* The delayed recognition of a sensory notion. Typically emotive-oriented **notions** process quickly, such as fear; while non-emotive **notions** may require significant time, such as the processing of a road map location.

*Attitudes:* The pattern of cognitive resources that provide context (level of importance) to a choice or behaviors. They are internalized notions that may have or not have emotive content.

*Behavior:* The externalized choice that will result in an action. It is the combination of a choice selection, triggering, and any intensification of the response due to existing circumstances (e.g., **notions, incongruity, or attitude**).

*Beliefs:* **Low-order** beliefs are the same as **notions**. **High-order** beliefs are the same as **attitudes**.

*Blueprint:* The fixed relationships (connections) among **cues, notions, expectations, incongruity, cognitive resources, intents, behaviors** and **social-networks**. They are the causal mapping of the key psychological elements of interest for a modeled **entity**. The blueprint shows the organization of **information** flow among and within the entities.

*Choice/Choice Set:* The set of possible outcomes (e.g. selections or behaviors) for a given set of input information (e.g., notion, incongruity) flows.

*Cognitive Resource:* In the model, the level of beliefs, internalized norms, memory, emotive sensitivity, or genetic propensity affecting a choice **selection** or **attitude**. It is reinforced through **conditioning** and diminished through atrophy.

*Conditioning:* The change in the level of a **cognitive resource** due to learning.

*Cues:* **Stimuli** that are salient to specific decision processes.

*Deeper:* The parsed set of implications (usually associated with higher-order beliefs beneath the superficial interpretation), also noted as the inner layers of information synthesis.

*Discrimination:* Pertaining to the intensity of **notions** after **assimilation** and **amplification**.

*Dominance:* Rather than selecting discrete motivations or beliefs, the model uses logic of dominance that converts rule based (logic) equations into continuous presentation with probabilistic interpretations. Dominance is defined as a critical degree of influence on the downstream process.

*Effective:* Pertaining to the remaining active **notions** after **prioritization**.

*Engine:* A collection of computational modules that represent the unified psychological theories able to simulate the behavioral dynamics of an **entity**. Each module contains the mathematics delineating a primal psychological theory. The modules are the components or building blocks of the engine. The engine executes (a simulation of) the **Blueprint** for the existing environment of **stimuli**.

*Entity:* Any specific parameterization of the **engine** with a **blueprint**. In the executing model, an entity represents the psychological responses from a specific individual or a specified collective of individuals.

*Evaluation:* The assessment of the utility of a choice based on **notions, expectations, incongruity, attitudes**, and other **selections**, any of which may represent emotive or non-emotive **information**.

*Excitation/Excitatory:* Activation pressure as a probability on the **triggering** of a **behavior** or on **conditioning** of **cognitive resources**. The level of arousal due to an internal or external **information** flow.

*Expectations:* The remembrance, averaging (filtering), or projection of historical **notions** for comparison to current **notions**.

*External Condition:* the computational representation of the physical world as affected by an **entity** actions. It generates physical signals (physical consequence) for **entities**. It is often part of a feedback process and links the cognitive entities to the physical world.

*Externally driven:* The component of psychological processes dominated by external (cue) information. Also see **External Condition**.

*Formation:* The recognition of a pattern based on information, as in **attitudes** and **notions**.

*High Level Cognition:* Mental processes that involve complex, composite concepts, such as a religious belief.

*High-order:* Complex patterns of psychological information such as a religious belief, a racial attitude, or the future personal-security implications of a current physical condition. See **high-Level Cognition**.

*Incongruity:* The discrepancy between **notions** and **expectations**. The incongruity may be perceived with differing intensity for **conditioning, evaluation, and behaviors**. At a **low-level cognition**, a notion could also be called dissonance.

*Indicated:* Pertaining to potential **behavior** that is only dependent on **selection** and **triggering** pressures.

*Information:* The processed representation of a **signal**.

*Inhibition/Inhibitory:* The restraint pressure as a probability on the **triggering** of a **behavior** or on **conditioning** of **cognitive resources**. The level of avoidance due to an internal or external **information** flow.

*Internally driven:* The component of psychological processes dominated by internal (attitude and cognitive-resource) information.

*Low-level Cognition:* Mental processes that involve simple constructs of sensory signals, such as the notion of "hot."

*Low-order:* Simple patterns of psychological information such as sensory notions or the belief that a physical condition exists. *See Low-level Cognition.*

*Notion:* The pattern of stimuli that invoke a specific concept relevant to choice **evaluation**, **conditioning**, or **behavior**. A notion may have or not have emotive content. At a **low-level cognition**, a notion could also be called a perception. Typically emotive-oriented notions contain much less information than non-emotive-oriented notions.

*Offset:* the variation that is required before a comparison of a current **notion** and the **expectation** of the notion are considered large enough to be recognized.

*Passivity:* The reduction or **offset** magnitude due to changes in the level of **cognitive resources**.

*Prioritization:* the limiting sensory or cognitive processing to stay below the maximum capability of the **entity**. The **discrimination** of **notions** and the **realization** of **behavior** are affected by the need to prioritize under intense **information** flow conditions.

*Qualitative Choice Theory:* The conditional probabilistic **selection** of a choice based on its utility from perceived **information** in combination with **entity** tastes and preferences.

*Realized:* Pertaining to actualized **behavior** that is the remaining **indicated behavior** after **prioritization**.

*Scaling:* An estimated parameter that normalizes (scales) a **notion** or **utility** to generate a "status-quo" value under "status-quo" conditions. The scaling reconciles any units of measure in the model to ensure that "normal" levels of stimuli produce the normal levels of behavior within the entity.

*Selection:* The preferred choice based on the perceived utility of each choice and the uncertainty of the utility. The selection will potentially become a **behavior**. At a **low-level cognition**, a selection could also be called an intent.

*Sensory:* Denoting fundamental experiential processing of **stimuli**, as in a sensory **notion**.

*Signal:* Any measurable phenomenon.

*Social network:* The connection paths for signals to/from **entities** and the **external conditions**; also noted signal dissemination.

*Stimuli:* External signals subconsciously or consciously recognizable by an entity.

*Superficial:* The basic physical statement of a condition, also noted as its outer or external feature.

*Tiering:* The process of using the same class of information (e.g. **selection**) to modify or select components within the same class (e.g. another **selection**).

*Trigger:* The combined level of response allowed by the competing influences of **excitation** and **inhibition**.

*Utility:* The perceived ordinal value of a choice in comparison to other choices. The utility used in the model is always ordinal (relative) rather than ordinal (absolute measure), because cardinal utility has no physical basis.

*Weight:* An estimated parameter that designates the importance of specific **information** content when making a choice or recognizing a pattern.

## Appendix 2: Mathematical Notes

The equations presented in computer code in the appendices that follow are based on the conventions of the PROMULA ADS simulation language (<http://promula.com/>).

1) Most arrays will be very sparse having only values for a few elements. Initially, most arrays will have no entries, and only a few will have one or two entries. We are trying to develop a flexible framework on one hand, but also simultaneously explore alternative theories and approaches on the other.

In this work, unless explicitly noted otherwise, if A, B and C are multidimensional arrays, then:

$$C = A \times B$$

or

$$C = A * B$$

is, for example with three indices,

$$C(i,j,k)=A(I,j,k)*B(I,j,k) \text{ over all indices, where } i=1 \dots I, j=1 \dots J, k=1 \dots K$$

2) When a set name is used explicitly in an equation other than one indexing a variable (e.g., Var1(set1, set2), three conventions are used:

Set1:m is the maximum index of the set; Set1:n is number of indices active, and Set1:s is the currently active index, for example in a “Do” loop. As an example: Var1=Set1:m sets Var1 to the number of entries for the set Set1.; Var1=Set1:n is the number of currently active indices in the set. See PROMULA Language Manual for other syntactical conventions (for example, the “Select” statement on indices that sets the range of the active indices).

3) To allow multiple calculation paths, the model uses a mathematical convention of:

$$\text{Var}=g*(1-\eta)+h*\eta$$

Where g and h are functions and  $\eta$  is a Boolean parameter to decide which function determines the value of the output variable (Var)

4) Maximum functions:

$\text{Var1}=\text{XMAX}(\text{Var2}(c))$  is the maximum value of VAR2 over the set c.

$\text{Var1}(r)=\text{MAX}(c)(\text{Var2}(r,c))$  is the maximum value of VAR2 over the set c, for each set r.

5) Note that “e” without a superscript is a (entity) set index. Otherwise it is the base of the natural logarithm.

6)  $\infty$  ( $\aleph$ =infinity) is numerically  $>10^{12}$ ;  $\epsilon$  (small) is numerically  $<10^{-45}$

7) Time subscripts: “Current” is the time at the current moment; “Prior” is the time at the previous moment; “Next” is the time of the upcoming moment.

8) The time concept of “Moment” is meant to allow various (real and absolute) length time units. If the time units is long relative to cognitive processes (e.g., partial day), then stimuli, for example, stimuli may immediately affect perceptions without delay. If the “moment” time unit is in seconds, then even what is normally considers instantaneous may contain a delay process through the assimilation process. The need to use a delay for perception is dependent on the stimulus and the simulated time unit for a specific notion.

9) The model is designed to allow initialization under transient conditions, but this means there are sufficient data to describe the state to the transient. More commonly, the model would begin a simulation in (real or assumed) equilibrium conditions. Typically, data exist to specify external conditions (stimuli cues) and historical average notions (perceptions of the stimuli). From these data, the other initials conditions are derivable.

10) We define stimuli, notions, selections, cognitive resources, referents, and behaviors as positive semi-definite. There are no negative versions of these entities. Differences in inhibitory versus excitatory conditions are captured by making additional entities which have positive semi-definite values, even if their existence negatively affects (pushes toward zero) another component of the cognitive process.

11) The time constants in the model strongly affects model stability. For example, rapidly changing expectations (small time constants for updating memories) can readily produce oscillations in behavior which may or may not be realistic, dependent on what historical data justifies. Similarly, long time constanst in converting entity behavior in the externally recognizable actions (stimuli) can also induce oscillatory behavior. (This process is considered the source of economic business cycles.)

12) Utility is commonly used in economic and physiological literature. We focus on it use within the context of QCT. The utility could be linearly additive  $u=\text{sum}(a*x)$ . In situations where the absolute level of a quantity is not as much of a concern and its relative level (e.g., 20% off the price



on an expensive item versus on a inexpensive item), the form could take  $u = \sum(a \cdot \ln(x))$ . In QCT the utility is used within a function of  $\exp(u)$ . In the logarithm case, this reduces to  $X^a$ . When developing a notion about the environment we directly use the  $X^a$  characterization. When applying the utility, we use the  $A \cdot X$  characterization because the  $x$  term used in the QCT is typically already comparing proportional change in notions.



## Appendix 3: Model Organization

### Data files.

The model needs a database for the raw data. These data are used to estimate model parameters and perform validation. That data set is used for parameterization and to generate a referent base-case input data set used as the basis for UQ and scenario analysis.

Arrays have the working indices in the model and will typically have 100-10000 elements. The “dt” will often be 1) on the order of 0.5 days (long term runs where conditioning is relevant) covering up to several years or 2) be on the order of seconds or minutes (short-term runs focusing on the response to rapidly changing conditions) that could cover a week. When run stochastically, the model would minimally generate 20 runs (the statistical lower limit for Latin Hypercube experiments) and often over 1000 (for quantified confidence levels). When in combination with UQ or intervention analysis, the number of runs would be thus again multiplied minimally by 20 and normally 100, but possibly (in “search” mode) by 1000. To limit data base size, each run should have its own configuration, input and output data base (in a separate directory) rather than extend the number of indices per variable. This organization is noted below. There is the mechanical issue that the “working” data base for a specific run needs to use a common alias (e.g. Input, Output) to keep the model code generic and transparent.

#### Define File

HData 'Experimental and entity data'

Config 'Information on the run and setup of model configuration'

BaseInput 'The data set that produces the "best estimate" nominal run'

Input# 'The complete input file for Scenario run #'

Output# 'The complete output file for Scenario run #'

The estimation and calibration process converts raw data into parameters and self-consistent (comparable) data sets. In modeling efforts, the reconciled historical data is often noted with the same variable name as the model data except with “x” (exogenous) suffix. Each parameter use in the model contains a mirror image with a “b” prefix designating it is a base or best-estimate value. The associated parameter name with a “r” prefix designates a metric on the range of uncertainty for the parameter. A third associated parameter with a “t” prefix defines the type of parameter and infers how to interpret the “range” – such a standard deviation on a Gaussian distribution or the +/- on a uniform distribution. The actual value of the parameter, as used in the model can vary when UQ, SA, and search analyses vary the parameters over their ranges of uncertainty.

## Model Sets and Global Parameters

In this discussion, a parameter is an exogenous data entity, often constant over time. A variable is a data entity calculated within the model and typically time-variant. Parameters come in two forms. Estimated parameters come from the statistical analysis of the raw data to specify the initialization and coefficients for equations. Scenario parameters are user-designated information about future conditions unaffected by the model. (They are outside the boundary of the model). They may include physical phenomena such as earthquakes or oil prices, or they can include any causal form of policy intervention. A set is the range of indices that a variable or a parameter includes. An index can be for the entity, a notion, a cognitive resource, a choice, or a stimulus.

The “Sets” define the indices on the model variables. Control parameters guide the model simulation and variable updating for differential equations. Numerical integration uses the simple first-order Euler approach. More sophisticated integration schemes would not be expected to improve the accuracy of model results, but can be used as desired.

### Define Set

c(6) ‘Choices’

d(6) ‘Cross Entities’

j(6) ‘Cross/Tiered Notions’

k(3) ‘Choice Class’

n(6) ‘Cognitive Resources’

q(6) ‘Cross/Tiered Choices’

r(6) ‘Entity’

t(1000) ‘Moment - time’

w(6) ‘Cross/Tiered Cognitive Resources’

y(6) ‘Notion’

z(6) ‘Cue’

\* placeholder variable subscripts: m =min or max value; x=exogenous”; f= activation; g inhibition;

\* h=upper (positive) value of incongruity; i lower Negative value of incongruity

\*

lruns(100) ‘LHS runs’

nruns (1000) ‘Stochastic runs’

### End Set

\*

\* Model Control Parameters

### Define Parameter

Current ‘Current Moment’

Next ‘Next Moment’

Prior ‘Prior Moment’

Lmax 'Number of LHS runs'

Nmax 'Number of scenario runs'

EMOMENT 'End-of-run Moment'

LMOMENT 'Last Moment'

SMOMENT 'Starting Moment'

\* Switch to designate deterministic results (winner take all)

Deterministic=0

\* Switch to designate probabilistic (stochastic)

Probabilistic=1

\* Switch to designate group-level results (fractional response)

Group=2

**End Parameter**

## Framework Control

Below is the "umbrella" framework defining the model organization. The User Interface(GUI) is a separate but linked task.

The configuration routines create the model to match the scope and blueprints of the model entities by linking components together. The estimation process contains the estimate errors in the parameter for use with UQ efforts. We can treat interventions as uncertain quantities and run the model to search the response-space for intervention portfolios that match user criteria. Initial post-processing is intended to use windowed partial correlation coefficients and confidence intervals (McKay 1976, Ford 2009). The model is normally treated as stochastic with a loop over multiple instantiations. The UQ loop is around the stochastic loop.

### Define Framework

- \* Interface wraps around framework parts  
    Call Interface
- \* Configure model size and characterize scenarios  
    Do Configure
- \* Manage raw data database  
    Do FillData
- \* Estimate equation coefficients with standard and residual error terms  
    Do Estimate
- \* Adjust constants (remove "error") to match "history"  
    Do Calibrate
- \* Update "Basecase" database  
    Do InData
- \* If formal UQ analysis, determine Latin-Hypercube experiment  
    Do LHS
- \* Run Simulations  
    Do Model
- \* Post-Process results to output files  
    Do OutData
- \* Generate statistics and visualizations for assessment  
    Do Visualization

### End Framework

The Sensitivity Analysis (SA) can be used to search for robust interventions. By robust we mean interventions that produce the desired outcome with a high probability of success despite model and data uncertainty – but only allowing limited theoretical [epistemic] uncertainty. This type

of analysis uses the LHS (Latin Hypercube Sampling) approach to search the intervention space for a portfolio of interventions (and their associated intensity) that best match mission criteria. If all interventions are independent, the LHS routine can sample the entire space unconditionally.

The LHS UQ (Uncertainty quantification) analysis is simply a sampling process:

Procedure LHS

Read Disk (B\_Param, R\_Param, T\_Param)

For a subset of the model Parameters:

Param =LHS as function of B\_Param and R\_Param with T\_Param characterization.

End Procedure

To maintain self consistency of the model characterization, the UQ requires conditional probabilities for parameter values. The remaining model parameters are then re-estimated from X-Var (raw data) to maintain consistency with entity characterizations.

Note the model is already stochastically based on a probability distribution for potential responses. The UQ represents a second-order uncertainty that is, however, difficult to interpret in context. It is an uncertainty on the uncertainty but it may change the statistical and dynamics results of the normal stochastic ensemble dramatically.

For sensitivity analysis we would typically vary one parameter at a time. We want to be able to designate particularly “interesting” runs and re-play them to determine sensitivity at specific transitional conditions. Other than for exploration, we do not want to re-play the model from a critical-transient point because a change in a parameter may prevent the point from occurring and such an analysis would produce inconsistent conclusions. Varying multiple parameters at once has the same conditional probability issues as the LHS analysis, except that here decomposing impacts across multiple parameter changes and interpreting the results require approaches not present in the conventional econometric/statistical literature. The DOE ASC program, implemented at SNL, has developed some techniques for this problem..

Advanced techniques would use a self-consistent manner to base the SA on model regimes. That is, what is a legitimate way to vary parameter once a model has changed behavioral modes? It may only be a particular set a parameter values that even allow the model to enter that region, thereby making any change to those parameters incompatible with the state of the system.

Scenario sampling is on the stochastic components of the model. All model results show a “confidence “envelope. We can treat interventions as uncertain and search for robust portfolio. Windowing calculates Partial Correlation Coefficients (PCC) in a moving time window to capture regime dependency.

## Model Execution

This is procedure that actually runs the simulation. If a short term run, bypass Attitude, Passivity, and Cognitive Resource dynamics.

### Define Procedure Model

- \* If Lmax is gt 1, then we are doing UQ, SA, or search.
- \* First run is always with best estimate values.
- Select Lrun(1-Lmax)
- \* Start outer Stochastic (K+LHS) loop on
  - Do Lrun
  - \* Determine values of model parameters.
    - Do LHS (Algorithms developed be V&V team)
    - Select Mrun(1-Rmax)
    - Do Mrun
  - \* Select time range from Starting moment to Ending moment.
  - \* This allows restart from any point in a stored past simulation.
    - Select Moment (SMoment-EMoment)
  - \* All differential equations need state variables initialized
    - Do Initialize
  - \* Start march over time for simulation
    - Do Moment
  - \* Current is the active moment in the loop
    - Current=Moment:s
    - Prior=XMAX(SMOMENT,Current-1)
    - Next=XMIN(LMOMENT,Current+1)
  - \* (Social network and physical interties) Map physical and entity actions to all affected entities
    - Do Stimuli
  - \* Determine impact of entity action on external environment.
    - Do External
  - \* Calculate Attitudes based on cognitive resources
    - Do Attitudes
  - \* Calculate passivity and Offsets based on cognitive resources
    - Do Passivity
  - \* Calculate Dissonance between current notions and expectation of conditions
    - Do Incongruity
  - \* Map patterns of Cue Stimuli into Notions
    - Do Notion
  - \* Create memory of notions for expectations of “normal” values
    - Do Expectation



- \* Reinforce referent memory intensity based on schema and perceptions
  - Do CogRes
- \* Decide choice intent
  - Do EvalSel
- \* Affect behavior based on Referents (Norms) and dissonance
  - Do Behavior
- \* Transform Behavior in physical consequence via the associated physical action
  - Do Action
- \* Map action of entities to external environment (Social Network)
  - Do Stimuli
- End Moment
- End Mrun
- End Lrun
- End Procedure Model**



## Appendix 4: Attitudes

Attitudes are a pattern of cognitive resources that provide context for notions, intents, and behaviors. Cognitive resources are those experiential and genetically derived reservoirs of cognitive and emotional conditions that act as the levels for beliefs, norms, memory, and knowledge. Collectively, with various weighting, they form the basis for our attitudes. Cognitive resources (see Appendix 9) are the internal (primal/atomistic) parts that we connect together (associate) to form attitudes. Thus, attitudes are composite entities. Attitudes are internalized perceptions.

If the model is run over a short time period (days) where learning is not significant, the updating of cognitive resources and therefore also the updating of attitudes and passivity (See Appendix 5) can be bypassed. In such instances it is best to set all of them to unity.

Attitude is used to amplify (increase or diminish) the importance of information on the choice and behavioral process. The equations in the model inherently allow for psychologically consistent changes in valence and in intensity.

### Define Variable Block

$N_B(r,c)$  Attitude for Behavior Intensity  
 $N_{Bf}(r,c)$  Attitude for Behavior Excitation  
 $N_{Bg}(r,c)$  Attitude for Behavior Inhibition  
 $N_{Dh}(r,y)$  Attitude for Upper Incongruity  
 $N_{Di}(r,y)$  Attitude for Lower Incongruity  
 $N_I(r,c)$  Attitude for Intent  
 $N_p(r,y)$  Attitude for Notion Intensity  
 $N_R(r,n)$  Attitude for Cognitive Resource Reinforcement  
 $N_{Rf}(r,n)$  Attitude for Cognitive Resource Excitation  
 $N_{Rg}(r,n)$  Attitude for Cognitive Resource Inhibition  
 $N_S(r,y,z)$  Attitude for Cue Importance to Notion  
 $R(r,n)$  Cognitive Resources  
End Variable Block

### Define Parameter Block

$h_B(r,n,c)$  Attitude weight for Behavior Intensity  
 $h_{Bh}(r,n,c)$  Attitude weight for Behavior from positive Incongruity  
 $h_{Bi}(r,n,c)$  Attitude weight for Behavior from negative Incongruity  
 $h_{Dh}(r,n,y)$  Attitude weight for Upper Incongruity  
 $h_{Di}(r,n,y)$  Attitude weight for Lower Incongruity  
 $h_I(r,n,c)$  Attitude weight for Intent

$\mathfrak{h}_P(r,n,y)$  Attitude weight for Notion Intensity  
 $\mathfrak{h}_R(r,n,n)$  Attitude weight for Cognitive Resource Reinforcement  
 $\mathfrak{h}_{Rh}(r,n,n)$  Attitude weight for Cognitive Resource from positive Incongruity  
 $\mathfrak{h}_{Ri}(r,n,n)$  Attitude weight for Cognitive Resource from negative Incongruity  
 $\mathfrak{h}_S(r,n,y,z)$  Attitude weight for Cue Importance to Notion  
 $\mathfrak{K}_B(r,c)$  Attitude scaling for Behavior Intensity  
 $\mathfrak{K}_{Bh}(r,c)$  Attitude scaling for Behavior from positive Incongruity  
 $\mathfrak{K}_{Bi}(r,c)$  Attitude scaling for Behavior from negative Incongruity  
 $\mathfrak{K}_{Dh}(r,y)$  Attitude scaling for Upper Incongruity  
 $\mathfrak{K}_{Di}(r,y)$  Attitude scaling for Lower Incongruity  
 $\mathfrak{K}_I(r,c)$  Attitude scaling for Intent  
 $\mathfrak{K}_P(r,y)$  Attitude scaling for Notion Intensity  
 $\mathfrak{K}_R(r,n)$  Attitude scaling for Cognitive Resource Reinforcement  
 $\mathfrak{K}_{Rh}(r,y)$  Attitude scaling for Cognitive Resource from positive Incongruity  
 $\mathfrak{K}_{Ri}(r,y)$  Attitude scaling for Cognitive Resource from negative Incongruity  
 $\mathfrak{K}_S(r,y,z)$  Attitude scaling for Cue Importance to Notion  
 End Parameter Block

Default: All  $\mathfrak{K}=1.0$  and all  $\mathfrak{h}=0.0$  to make all  $N=1$

If the model is started in a transient condition, then it probably beneficial to set  $N$  initially to unity by setting  $\mathfrak{K}$  to:

$$\mathfrak{K} = \mathbf{1} / \prod_n R^{\mathfrak{h}}$$

The Attitude is based on existing Cognitive resources (noted numerically as the prior condition)

The  $\mathfrak{K}$  is a scaling term ensures the attitude ( $N$ ) is normalized to unity or any other user-defined value as the initial condition. The  $\mathfrak{h}$  is the weighting of each cognitive resources ( $R$ ) defining the attitude ( $N$ ).

### Define Procedure Attitude

Select Moment (Prior)  
 Read Disk ( $R$ )  
 Select Moment (Current)

\*The attitude associated with physical sensory input (i.e. cuing stimuli placed in context) is  $N_s$ .

$$N_S(r, y, z) = \kappa_S(r, y, z) \times \prod_n R(r, n)^{h_S(r, n, y, z)}$$

\*The attitude toward the importance of a notion the pattern of sensory inputs generate is  $N_P$ .

$$N_P(r, y) = \kappa_P(r, y) \times \prod_n R(r, n)^{h_P(r, n, y)}$$

\*Incongruity is a variance between the level of sensory notion we are experiencing and our expectation of what level that notion should be. Incongruity is a low level form of dissonance.  $N_D$  is the attitude for incongruity. There is a different attitude for positive and negative incongruity. That is, individual may see “worsening” conditions with a different concern than for “improving” conditions.

$$N_{Dh}(r, y) = \kappa_{Dh}(r, y) \times \prod_n R(r, n)^{h_{Dh}(r, n, y)}$$

$$N_{Di}(r, y) = \kappa_{Di}(r, y) \times \prod_n R(r, n)^{h_{Di}(r, n, y)}$$

\*Similarly there are attitudes ( $N_R$ ) toward conditioning (learning) to reduce improved cognitive resources to resolve incongruity. There are attitudes ( $N_I$ ) toward preferences in evaluation and making choices (intents) , and attitudes ( $N_B$ ) for when to respond outwardly with a behavior.

$$N_R(r, n) = \kappa_R(r, n) \times \prod_w R(r, w)^{h_R(r, n, w)}$$

\*In addition to core attitudes toward cognitive resources (R), there are sub elements that are concerned with the conditioning for specific resources depending on the incongruity.

$$N_{Rh}(r, n) = \kappa_{Rh}(r, y) \times \prod_w R(r, w)^{h_{Rh}(r, n, w)}$$

$$N_{Ri}(r, n) = \kappa_{Ri}(r, y) \times \prod_w R(r, w)^{h_{Ri}(r, n, w)}$$

\*The "w" indices in the above equation act to capture the interaction of all cognitive resources with other cognitive resources.

\*There are attitudes ( $N_I$ ) toward preferences in evaluation and making choices (intents), and attitudes ( $N_B$ ) for when to respond outwardly with a behavior.

$$N_I(r, c) = \mathcal{K}_I(r, c) \times \prod_n R(r, n)^{\mathfrak{h}_I(r, n, c)}$$

$$N_B(r, c) = \mathcal{K}_B(r, c) \times \prod_n R(r, n)^{\mathfrak{h}_B(r, n, c)}$$

In addition to core attitudes toward triggering behavior (B), there are sub elements that are concerned with the activation (f) and restriction (g) on specific behaviors.

$$N_{Bh}(r, c) = \mathcal{K}_{Bh}(r, c) \times \prod_n R(r, n)^{\mathfrak{h}_{Bh}(r, n, c)}$$

$$N_{Bi}(r, c) = \mathcal{K}_{Bi}(r, c) \times \prod_n R(r, n)^{\mathfrak{h}_{Bi}(r, n, c)}$$

Write Disk( $N_B, N_{Bf}, N_{Bg}, N_{Dh}, N_{Di}, N_I, N_p, N_R, N_{Rf}, N_{Rg}, N_S$ )

**End Procedure**

## Appendix 5: Passivity and Offsets

Offsets determine how much incongruity is needed before it is noticed and acted upon. Passivity modifies the offset by increasing or decreasing its size. The use of this variable produces only secondary effects and can typically be neglected. As such, the default is for all  $\xi=1$ , and  $\zeta=0$ , so that  $E=1$ .

Select Moment (Prior)

Read Disk (R)

Select Moment (Current)

\*Passivity (E) affects the level of incongruity needed before an entity recognizes it as a concern and potential requiring a response.

\*The equations for passivity to positive and negative incongruity toward notions, respectively, are shown below.

$$E_{Ph}(r, y) = \xi_{Ph}(r, y) \times \prod_n R(r, n)^{\zeta_{Ph}(r, n, y)}$$

$$E_{Pi}(r, y) = \xi_{Pi}(r, y) \times \prod_n R(r, n)^{\zeta_{Pi}(r, n, y)}$$

\*The passivity toward conditioning (learning) in the face of positive or negative incongruity is calculated in the following equations.

$$E_{Rh}(r, n) = \xi_{Rh}(r, n) \times \prod_w R(r, w)^{\zeta_{Rh}(r, n, w)}$$

$$E_{Ri}(r, n) = \xi_{Ri}(r, n) \times \prod_w R(r, w)^{\zeta_{Ri}(r, n, w)}$$

\*The passivity toward behavioral triggering, as it depends on the level of positive or negative incongruity, is calculated in the two equations below.

$$E_{Bh}(r, c) = \xi_{Bh}(r, c) \times \prod_N R(r, n)^{\zeta_{Bh}(r, n, c)}$$

$$E_{Bi}(r, c) = \xi_{Bi}(r, c) \times \prod_N R(r, n)^{\zeta_{Bi}(r, n, c)}$$

\*The Offset (percentage discrepancy -- O) is the modification to the initial discrepancy ( $O_0$ ) Through the passivity multiplier.

\*The offsets for positive and negative incongruity toward notions are:

$$\begin{aligned} O_{Ph}(r, y) &= O_{Ph0}(r, y) \times E_{Ph}(r, y) \\ O_{Pi}(r, y) &= O_{Pi0}(r, y) \times E_{Pi}(r, y) \end{aligned}$$

\*The offsets for positive and negative incongruity toward Cognitive Resource reinforcements/conditioning are:

$$\begin{aligned} O_{Rh}(r, n) &= O_{Rh0}(r, n) \times E_{Rh}(r, n) \\ O_{Ri}(r, n) &= O_{Ri0}(r, n) \times E_{Ri}(r, n) \end{aligned}$$

\*The offsets for positive and negative incongruity toward Behavioral triggering are:

$$\begin{aligned} O_{Bh}(r, c) &= O_{Bh0}(r, c) \times E_{Bh}(r, c) \\ O_{Bi}(r, c) &= O_{Bi0}(r, c) \times E_{Bi}(r, c) \end{aligned}$$

\*The Offset for sensory perception is assumed to be biological and arbitrarily small.

$$O_S(r, y) = \varepsilon * 1e6$$

Write Disk( $E_{Bh}$ ,  $E_{Bi}$ ,  $E_{Ph}$ ,  $E_{Pi}$ ,  $E_{Rh}$ ,  $E_{Ri}$ ,  $O_{Bh}$ ,  $O_{Bi}$ ,  $O_{Ph}$ ,  $O_{Pi}$ ,  $O_{Rh}$ ,  $O_{Ri}$ )

End Procedure



## Appendix 6: Incongruity

Unlike stimuli and notions, incongruity takes on both positive and negative values. However, incongruities, such as those associated with overwork in a job or too little work (unemployment) are passed on as two different notion flows, even if the input stimuli to the notions overlap. The offset is the difference that maximizes the attention to the notions and its use in other cognitive processes.

Note that there may be multiple incongruities for the same notion. If the cue is noise, there can be a comparison with the remembered silence or a set volume (e.g., the last ear-pain of a rock concert). The expectations (e.g., expected silence or expected sound intensity) can then be compared for use in a utility function specific to the choice involved.

### Define Variable Block

$D_{Bh}(r,c)$  Positive Dissonance for Behavior

$D_{Bi}(r,c)$  Negative Dissonance for Behavior

$D_{Ph}(r,y)$  Positive Dissonance for Evaluation

$D_{Pi}(r,y)$  Negative Dissonance for Evaluation

$D_{Rh}(r,n)$  Positive Dissonance for Conditioning Cog. Resources

$D_{Ri}(r,n)$  Negative Dissonance for Conditioning Cog. Resources

$H(r,y)$  Expectations

$O_{Bh}(r,c)$  Offset toward Positive Dissonance for Behavior

$O_{Bi}(r,c)$  Offset toward Negative Dissonance for Behavior

$O_{Ph}(r,y)$  Offset toward Positive Dissonance for Evaluation

$O_{Pi}(r,y)$  Offset toward Negative Dissonance for Evaluation

$O_{Rh}(r,n)$  Offset toward Positive Dissonance for Conditioning Cog. Resources

$O_{Ri}(r,n)$  Offset toward Negative Dissonance for Conditioning Cog. Resources

$O_s(r,y)$  Offset for Sensory Cues

$PP(r,y)$  Assimilated Notion

$R(r,n)$  Cognitive Resource

End Variable Block

### Define Parameter Block

$\alpha_{Bh}(r,c,y)$  Positive Incongruity Scaling for Behavior

$\alpha_{Bi}(r,c,y)$  Negative Incongruity Scaling for Behavior

$\alpha_{Dh}(r,n,y)$  Positive Incongruity Scaling for Cog. Resources

$\alpha_{Di}(r,n,y)$  Negative Incongruity Scaling for Cog. Resources

End Parameter Block

Default  $\alpha=0$

Define Procedure Incongruity

\*

Select Moment (Prior)

Read Disk (PP,H)

Select Moment (Current)

\*

\* The incongruity is the difference between a perception and the accepted range of a expectation (via an offset). There is a lower limit of sensory realization  $O_s$ .

\*The "max" function selects the largest element in the vector field specified by the index.

$$D_{Ph}(r, y) = xmax \left( 0, \frac{[PP(r, y) - H(r, y) \times (1 - O_{Ph}(r, y))]}{xmax(H(r, y), O_s(r, y))} \right)$$

$$D_{Pi}(r, y) = xmin \left( 0, \frac{[PP(r, y) - H(r, y) \times (1 + O_{Pi}(r, y))]}{xmax(H(r, y), O_s(r, y))} \right)$$

\* The “worst” notion drives behavioral dissonance.

$$D_{Bh}(r, c) = max(y)(\alpha_{Bh}(r, c, y) * \frac{[PP(r, y) - H(r, y) \times (1 - O_{Bh}(r, c))]}{xmax(H(r, y), O_s(r, y))})$$

$$D_{Bi}(r, c) = min(y)(\alpha_{Bi}(r, c, y) * \frac{[PP(r, y) - H(r, y) \times (1 + O_{Bi}(r, c))]}{xmax(H(r, y), O_s(r, y))})$$

\* Incongruity assessment compares the condition (notions) to the capability to deal with the condition.

$$D_{Rh}(r, n) = max(y) \left( \frac{[\alpha_{Rh}(r, n, y) * PP(r, y) - R(r, n) \times (1 - O_{Rh}(r, n))]}{R(r, n)} \right)$$

$$D_{Ri}(r, n) = min(y) \left( \frac{[\alpha_{Ri}(r, n, y) * PP(r, y) - R(r, n) \times (1 + O_{Ri}(r, n))]}{R(r, n)} \right)$$

\*

Write Disk( $D_{Bh}$ ,  $D_{Bi}$ ,  $D_{Ph}$ ,  $D_{Pi}$ ,  $D_{Rh}$ ,  $D_{Ri}$ )

**End Procedure**

## Appendix 7: Notions

Notions are a pattern of cuing stimuli that act as the lower levels of (primal) perception. The context for these perceptions is based on attitudes.

We use a product formulation for notion formation because a summation approach can inconsistently allow any information or resources to artificially dominant and it does not necessarily keep the primary information (e.g., a notion) the fundamental determinant of a response presumably caused by immediate cues. That is, the summation formulation could cause an inconsistent reversal of valence independent of the valence associated with the notion in any particular circumstance. The product logic does not have this problem and it is consistent with the psychology.

In equations, the term  $\alpha \prod R^{\beta}$  reflects the combination and contribution to how intensely a Notion, Incongruity, etc is applied to a process. It is also the pattern of cognitive resources that gives context to the Notions, Incongruities, etc. via attitudes.

The Assimilation process takes time with affective notion realization occurring faster than for reasoned notions. The rise in affective notion can exceed the threshold for a response and act to trigger behavior that might not otherwise occur if cognitive process dominated or timing were different. The lingering of an affective notion due the "afterimage" phenomenon of the assimilation delay in essence sets the mood of the entity. Because incongruity is the relationship between the assimilated notion and its expected values, the assimilation process can cause a delayed buildup of incongruity. The incongruity can reach a behavioral threshold some time after the actual initiating stimuli.

We use the concept of prioritization to accommodate sensory input overload. The most important notions (low level perceptions) as determined by discriminated intensity obtain the lion's share of attention.

In the equations below, the  $U_S$  is utility of the notion It determines the amplification of the perception from the stimuli. The first term in  $U_S$  considers the sensitivity of the person to the general perception. The second term considers any reinforcing or inhibiting stimuli. The third term considers reinforcing or inhibiting "memories" via incongruity. The forth term considers the tiering of other incongruities, where one notion (its incongruity) "activates" the importance of another notion.

With the use of decision field theory (Busemeyer 1993, 2002) to determine assimilated notions, the model can be run with time intervals down to seconds, or it can be run with multi-hour intervals. A key feature is that some notions arrive sooner than others (such as affective ones),

and if decisions need to be made promptly they may be different than what would occur without the time pressure. The assimilation process automatically capture the ideas of recency by remembering the previous events consistent with theory; it does not remember the duration and only remembers the maximum intensity. The notion takes time to die away and can affect future decisions. For affective notions, this response reflects the concept of moods. In the model, a discriminated notion is essentially a mood.

The concept of psychological recency is the lingering remembrance of a notion (Lerner 2004). The psychological concept of frequency defines the sensitivity of the reinforcement (cognitive resources) to incongruity (Perugini 2001). DFT allows ex post construction/rationalization. Consistent with experimental data, the mathematical representation of DTF in the model does not “notice” a smaller irritation if there is an active larger irritation. Stimuli are not additive and there is no build up.

QCT for a binary choice takes the form of  $1/(1+\exp(-u))$ . This form may often be used as an approach to a maximum level a saturation (or amplification) in regard to some stimuli. However, we are typically either concerned with response when the stimuli are far below a maximally tolerable value and this function form simplifies to  $\exp(u)$ , as in the amplification term for a Discriminated Notion. Additionally, in situations where notions rise to excessive levels, the continued use of  $\exp(u)$  merely generates another response to the extreme event faster. The uncertainty in response time is also large relative to the more-of-concern character of the response, thereby making the use of the more complicated representations (from a simulation and parameterization perspective) unnecessary.

If only one  $\alpha_p$  is non-zero, the stimulus is the Notion. We can use such an approach in the extreme of very limited data.

#### Define Variable Block

$D_{Ph}(r,y)$  Positive Dissonance for Evaluation

$D_{Pi}(r,y)$  Negative Dissonance for Evaluation

$M_p(r,y)$  Marginal Probability for Perception

$N_{Dh}(r,y)$  Attitude for Upper Incongruity

$N_{Di}(r,y)$  Attitude for Lower Incongruity

$N_p(r,y)$  Attitude for Notion Intensity

$N_s(r,y,z)$  Attitude for Cue Importance to Notion

$P_D(r,y)$  Discriminated Notion

$P_E(r,y)$  Effective Notion

$P_P(r,y)$  Assimilated Notion

$P_S(r,y)$  Sensory Notion  
 $S(r,z)$  Stimuli/Cue  
 $U_P(r,y)$  Prioritization Utility for Notion  
 $U_S(r,y)$  Amplification Utility for Notion  
 $\Phi_P(r,y)$  Utility of Notion  
 $\Omega_P(r)$  Total Utility of Notion  
 End Variable Block

#### Define Parameter Block

$L_P(r)$  Limit for total Notion Load  
 $\alpha_P(r,y)$  Notion amplification from Cognitive Resources  
 $\alpha_S(r,y)$  Cue Scaling for Sensory Notion  
 $\beta_P(r,y,j)$  Notion amplification from other cues  
 $\beta_S(r,y,z)$  Cue weight for Sensory Notion  
 $\gamma_{Ph}(r,y,j)$  Notion amplification from positive incongruity  
 $\gamma_{Pi}(r,y,j)$  Notion amplification from negative incongruity  
 $\mu_P(r,y)$  Prioritization Weight  
 $\tau_P(r,y)$  Decay time for notion  
 $\tau_S(r,y)$  Build-up time constant for notion  
 End Parameter Block

Default  $L_P=\infty$ ;  $\alpha_P, \beta_P, \gamma_P=0.0$ ;  $\alpha_S=1.0$ ;  $\beta_S=0.0$ ;  $\mu=2.0$ ;  $\tau=1.0$

#### Define Procedure Notion

\*

Select Moment (Prior)  
 Read Disk (PP)  
 Select Moment (Current)

#### \* Sensory Notion

$$P_S(r,y) = \alpha_S(r,y) * \prod_z S(r,z)^{\beta_S(r,y,z) * N_S(r,y,z)}$$

#### \* Assimilated Notion

\* De-noising also occurs here

$$P_P(r,y) = P_P(r,y) + dt \times \left( \frac{\text{xmax}\left(0, (P_S(r,y) - P_P(r,y))\right)}{(\text{xmax}(dt, \tau_S(r,y)))} - P_P / \text{xmax}(2 \times dt, \tau_P(r,y)) \right)$$

**\* Discriminated Notion**

\* The utility of recognizing these stimuli as a notion

$$U_S(r, y) = \alpha_P(r, y) \times N_P(r, y) + \sum_j \beta_P(r, y, j) \times N_P(r, j) \times P_P(r, j) +$$

$$\sum_j \gamma_{Ph}(r, y, j) \times N_{Dh}(r, j) \times D_{Ph}(r, j) + \sum_j \gamma_{Pi}(r, y, j) \times N_{Di}(r, j) \times D_{Pi}(r, j)$$

\* Strength of Discriminated Notion

$$P_D(r, y) = P_P(r, y) \times e^{U_S(r, y)}$$

\* Notion Prioritization; Limit intensity of all notions to be less than maximum sensory load

\* Notion Utility

$$U_P(r, y) = \mu_P(r, y) \times P_D(r, y)$$

\* Prioritization Weight

$$\Phi_P(r, y) = e^{U_P(r, y)}$$

\*Total Prioritization

$$\Omega_P(r) = \sum_y \Phi_P(r, y)$$

\* Allocation of Priorities

$$M_P(r, y) = \Phi_P(r, y) / \Omega_P(r)$$

\* Effective Notion

$$P_E(r, y) = \text{xmin}(L_P(r) \times M_P(r, y), P_D(r, y))$$

Write Disk( $U_S, U_P, P_S, P_P, P_D, P_E, M_P, \Phi_P$ )

**End Procedure**

## Appendix 8: Expectations

The formulation is based on Sterman 2000 as modified by Backus 2006. Expected values are solely based on remembered notions so the link is to the notion, not the stimulus. Expectation formation based on historical experience is not a function of the Cognitive Resource.

Define Variable Block

$A_P(r,y)$  Remembered Notion

$A_S(r,y)$  De-noised Notion

$G(r,y)$  Expected Notion Change rate

$H(r,y)$  Expectation

$P_P(r,y)$  Assimilated Perception

End Variable Block

Define Parameter Block

$G_0(r,y)$  initial growth rate

$H_0(r,y)$  Initial(average) expectation (Growth=0.0)

$\eta_H(r,y)$  Boolean for Averaging(0) or Forecasting(1)

$\tau_D(r,y)$  De-noise smoothing time

$\tau_E(r,y)$  Long-term Memory Adjustment Time

$\tau_g(r,y)$  Forecast time

End Parameter Block

Default  $\eta_H=0$ ,  $\tau_D=1.0$ ,  $\tau_E=1000.0$ ,  $\tau_g=0.0$ ,  $G_0=0.0$

$H_0$  will need case-by-case consideration

The state variables can be initialized with a historical growth rate ( $G_0$ ) – assumed to be 0.0 in most instances. Equations below based on Sterman (2000).

$$A_S(r,y) = P_P(r,y)/(1 + G_0(r,y) \times \tau_D(r,y))$$

$$A_P(r,y) = A_S(r,y)/(1 + G_0(r,y) \times (\tau_E(r,y) - \tau_D(r,y)))$$

But generally assume equilibrium start-up where  $A_S=A_P=H_0$

Define Procedure Expectation

\*

\* Remember the filtered Perception, not the stimuli

\*

Select Moment (Prior)

Read Disk( $P_p, A_s, A_p$ )

Select Moment (Current)

\*

\* Short-term memory of De-noised perception

$$A_s(r, y) = A_s(r, y) + dt \times (P_p(r, y) - A_s(r, y)) / \tau_D(r, y)$$

\* Long term memory of perception

$$A_p(r, y) = A_p(r, y) + dt \times (A_s(r, y) - A_p(r, y)) / (\tau_E(r, y) - \tau_D(r, y))$$

\* Perceived change rate in perception

$$G(r, y) = (\frac{A_s(r, y)}{A_p(r, y)} - 1) / (\tau_E(r, y) - \tau_D(r, y))$$

\* Remembrance is just memory or expectation

$$H(r, y) = A_s(r, y) \times (1 - \eta_H(r, y)) + A_s(r, y) \times (1 + G(r, y) \times \tau_s(r, y)) \times e^{(G(r, y) \times (\tau_g(r, y)))} \times \eta_H(r, y)$$

\*

Write Disk( $A_s, A_l, G, H$ )

**End Procedure**



## Appendix 9: Cognitive Resources

Cognitive resource is a broad term reflecting a learned capability for responding to notions (patterns of relevant cues). A pattern of cognitive resources represent an attitude. The attitude may have a reasoned or emotive basis and it can reflect a propensity for response or perceptions of which an entity is not consciously aware.

Anything learned (knowledge, belief, emotional response, intuition) other than pure memory of past conditions for making expectations, is a cognitive resource

The model dynamics indicate that “motivation” is the circumstance whereby perceptions are large enough to offer a challenge, yet small enough to ensure adequate response with readily achievable effort (Grossberg 1987, Yerkes 1908). This aspect is reflected as the excitatory and inhibitory components of conditioning.

The Cognitive Resources include belief, knowledge, experience, and emotive levels of memory to decisions. The current logic is based on coping skill dynamics (Backus 2006).

There is no problem with the model having mutually exclusive Cognitive Resources (Norms/Beliefs), such as a stereotypical Middle Eastern Muslim having both “Hate America” and “Love America” perspectives and feelings.

The model automatically exhibits learned helplessness dynamics as well as inattention (notions failing to produce adequate dissonance), and being overwhelmed (notion produce excess dissonance outside the range of effective behaviors). See Backus (2006) reference.

One  $R [R(r,1)]$  is a numeraire, arbitrarily set to a value and held constant. It could be said to represent the evolutionary (biological) behavioral referent of human nature. To some extent this is numerically expedient to reduce the number of constants in the model when we need to assume (for lack of data) that the cognitive resources are not a dynamic influence on every cognitive process.

Define Variable Block

$D_{Ph}(r,y)$  Positive Dissonance for Evaluation

$D_{Pi}(r,y)$  Negative Dissonance for Evaluation

$D_{Rh}(r,n)$  Positive Dissonance for Conditioning Cog. Resources

$D_{Ri}(r,n)$  Negative Dissonance for Conditioning Cog. Resources

$F(r,n)$  Intensity of Conditioning

$N_{Dh}(r,y)$  Attitude for Upper Incongruity

$N_{Di}(r,y)$  Attitude for Lower Incongruity

$N_p(r,y)$  Attitude for Notion Intensity  
 $N_R(r,n)$  Attitude for Cognitive Resource Reinforcement  
 $N_{Rh}(r,n)$  Attitude for Cognitive Resource from positive Incongruity  
 $N_{Ri}(r,n)$  Attitude for Cognitive Resource from negative Incongruity  
 $P_E(r,y)$  Effective Notion  
 $R(r,n)$  Cognitive Resources  
 $U_R(r,n)$  Utility for Intensity Conditioning  
 $U_{Rf}(r,n)$  Excitation utility for conditioning  
 $U_{Rg}(r,n)$  Inhibition utility for conditioning  
 End Variable Block

Define Paramter Block

$\alpha_F(r,n)$  Conditioning intnesity from self-reinforcement  
 $\beta_F(r,n,y)$  Conditioning intensity from notion intensity  
 $\gamma_{Fh}(r,n,y)$  Conditioning Intensity from Positive Incongruity  
 $\gamma_{Fi}(r,n,y)$  Conditioning intensity from Negative Incongruity  
 $\gamma_{Rh}(r,n,y)$  Conditioning Excitation from Positive Incongruity  
 $\gamma_{Ri}(r,n,y)$  Conditioning Excitation from Negative Incongruity  
 $\delta_{Rh}(r,n,y)$  Conditioning Inhibition from Positive Incongruity  
 $\delta_{Ri}(r,n,y)$  Conditioning Inhibition from Negative Incongruity  
 $\lambda_F(r,n,n)$  Conditioning Saliency from other cog. resources  
 $\tau_F(r,n)$  Time constant on Cog Resources formation ( $\infty$ =Default)[5=testing]  
 $\tau_R(r,n)$  Time constant on Cog Resources atrophication ( $\infty$ =Default)[100=testing]  
 End Parameter BLock

Default  $\tau_F=\infty$  [5=testing];  $\tau_R=\infty$  [100=testing];  $\alpha, \beta, \gamma, \delta=0.0$

Initial:  $N=\max(*1e6, \max(y)(H*(1+Op)))$ ,  $N(r,1)=1.0$  biological (or maybe all =1.0 will work)

### Define Procedure CogRes

\* Cognitive Resource Formation  
 Select MOMENT (PRIOR)  
 Read Disk(R)  
 Select MOMENT (CURRENT)

\* Utility of increasing the Cognitive Resourceis based on the Cognitive Resource and current notions.

The first term is the importance of R itself bas on other R; the  $N_P$  is that associated with importance of  $P_E$ ;  $N_D$  is that associated with importance of  $D_P$ . The parameters indicate the importance to the total utility.

$$U_R(r, n) = \alpha_F(r, n) \times N_R(r, n) + \sum_y \beta_F(r, n, y) \times N_P(r, y) \times P_E(r, y) +$$

$$\sum_y \gamma_{Fh}(r, n, y) \times N_{Dh}(r, y) \times D_{Ph}(r, y) + \sum_y \gamma_{Fi}(r, n, y) \times N_{Di}(r, y) \times D_{Pi}(r, y) +$$

$$\sum_w \lambda_F(r, n, w) \times N_R(r, w) \times R(r, w)$$

\* Utility of incongruity on Cognitive Resource reinforcement uses other Cognitive Resourcea to widen or narrow the range where incongruity affects reinforcement (conditioning).

\* Utility of incentives for learning is one component of incongruity and the Cognitive Resource's effect on Cognitive Resource conditioning.

$$U_{Rf}(r, n) = \max(y)(\gamma_{Rh}(r, n, y) \times N_{Rh}(r, y) \times D_{Rh}(r, y) + \gamma_{Ri}(r, n, y) \times N_{Ri}(r, y) \times D_{Ri}(r, y))$$

\* Utility of avoidance of learning is the other component of incongruity and Cognitive Resource's effect on behavioral triggering.

$$U_{Rg}(r, n) = \max(y)((\delta_{Rh}(r, n, y) \times N_{Rh}(r, y) \times D_{Rh}(r, y) + \delta_{Ri}(r, n, y) \times N_{Ri}(r, y) \times D_{Ri}(r, y)))$$

\* Reinforcement builds on existing the Cognitive Resource when there is incongruity, with context-based intensity

$$F(r, n) = R(r, n) \times \frac{e^{-U_R(f, n)}}{\tau_F(r, n)} \times \frac{1}{1 + e^{U_{Rf}(r, n)}} \times \frac{1}{1 + e^{U_{Rg}(r, n)}}$$

\* Reduction in a Cognitive Resource is due to atrophy ( $R/\tau$ ) only.

$$R(r, n) = R(r, n) + dt \times (F(r, n) - \frac{R(r, n)}{\tau_R(r, n)})$$

\*

Write Disk ( $U_R$ ,  $U_{Rg}$ ,  $U_{Rf}$ ,  $F$ ,  $R$ )

**End Procedure**



## Appendix 10: Evaluation and Selection

Choice evaluation and selection is based on QCT. One choice can trigger other choice sequences in a tiering process. Notions and decisions can be a cascading sequence of other perceptions and decisions in a process herein called Tiering. Tiering increases (amplifies) the probability of the choice. The tiering logic generalizes the intermediate goal logic (See the discussion on SHERCA in Chapter 2) and adds parallel intermediate perception logic. An entity may be parameterized to like cars, but if a suicide bomb goes off in front of it, the entity may not perceive the type of car that just flew over its head. This new cue will also change the intent evaluation process to, for example, “Where is a barrier to hide behind?”

Group choice is defined as fractions. Individual choice is a probability. It can remain stochastic or be deterministic. (Deterministic usage is primarily for diagnostic purposes.)

In a utility function, the sign of the constant term sets the valance, while the attitude and actual information (e.g., cues/stimuli) set the intensity (Lerner 2000).

Define Variable Block

$D_{Ph}(r,y)$  Positive Incongruity for Notions

$D_{Pi}(r,y)$  Negative Incongruity for Notions

$I(r,c)$  Evaluated Choice

$M_I(r,c,q)$  Marginal Probability of Choice

$N_{Dh}(r,y)$  Attitude toward positive Incongruity

$N_{Di}(r,y)$  Attitude toward negative Incongruity

$N_I(r,c)$  Attitude toward choice

$N_P(r,c)$  Attitude toward perception

$P_E(r,y)$  Effective Notion

$U_I(r,c)$  Utility of evaluated choice

$Y(r,q)$  Choice with maximum likelihood

$\Phi_I(r,c,q)$  Weighted Utility of Choice

$\Omega_I(r,q)$  Sum of weighted utility of choices

$\Pi_I(r,c,q)$  Cumulative probability of choices

End Variable Block

Define Parameter Block

$L_q$  Number of Choice Sets

$\alpha_I(r,c)$  Saliency of attitude on predisposition utility of choice.

$\beta_I(r,c,y)$  Saliency of Notions on utility of a choice.

$\gamma_{Ih}(r,c,y)$  Saliency of Positive Incongruity on utility of choice

$\gamma_{Ii}(r,c,y)$  Saliency of Negative Incongruity on utility of choice

$\varepsilon$  A small number

$\eta_c(r)$  Switch to decide stochastic, group, or winner-gets-all logic

$\lambda_I(r,c,q)$  Saliency of other choices on utility of current choice

$\sigma$  Random number (Uniform, 0,1)

$\Psi_I(r,c,q)$  Map of choice set to choices

End Parameter Block

Default  $L_q=1$ ;  $\alpha_I, \beta_I, \gamma_{Ih}, \gamma_{Ii}, \lambda_I=0.0$ ;  $\eta_c = \text{“Group”}$ ;  $\Psi_I=0.0$ ,  $\Psi_I(\text{diagonal})=1.0$

### Define Procedure EvalSel

Select Moment(Prior)

\*Read Disk()

Select Moment(Current)

\*

\* The utility of the choice depends on notions, incongruity and other selections, as

\* amplified by attitudes.

\* A Selection must be calculated before use and no simultaneity.

$$\begin{aligned} U_I(r, c) = & \alpha_I(r, c) \times N_I(r, c) + \sum_y \beta_I(r, c, y) \times N_P(r, c) \times P_E(r, y) + \\ & \sum_y \gamma_{Ih}(r, c, y) \times N_{Dh}(r, y) \times D_{Ph}(r, y) + \sum_y \gamma_{Ii}(r, c, y) \times N_{Di}(r, y) \times D_{Pi}(r, y) \\ & + \sum_q \lambda_I(r, c, q) \times N_I(r, q) \times I(r, q) \end{aligned}$$

\* Select choices if part of set  $\psi$

Select  $q(1-L_q)$

\* The availability and evaluation of the selection

$$\Phi_I(r, c, q) = \Psi_I(r, c, q) \times e^{(U_I(r, c))}$$

\* The total of utility weights

$$\Omega_I(r, q) = \sum_c \Phi_I(r, c, q)$$

\* Marginal probability of the selection

$$M_I(r, c, q) = \Phi_I(r, c, q) / \Omega_I(r, q)$$

\* Make cumulative distribution of choice probabilities.

\* (There is an implicit automatic loop here over 'c')

$$J_I(r, 1, q) = M_I(r, 1, q)$$

$$J_I(r, c, q) = J_I(r, c-1, q) + M_I(r, c, q)$$

$$Y(r, q) = \max(c)(M_I(r, c, q))$$

```

 $\sigma = \text{Random}(\text{Uniform}, 0,1)$ 
* Check for a group entity
Do If  $\eta_c(r)$  EQ Group
* The marginal probability becomes the fraction of group with that Intent
*  $M_I$  is now mutually exclusive so that all cross elements are zero to all the summation below
     $I(r,c)=\text{sum}(q)(M_I(r,c,q))$ 
End Do if Group
DO r
    DO q
        Select c*
    * If individual check if scenario is deterministic (diagnostic mode)
        Do If  $\eta_c(r)$  EQ Determinisitic
    * Select winning choice
        Select c if  $M_I(r,c)$  EQ  $\Upsilon(r,q)$ 
    * Winner take all
         $I(r,c)=1$ 
        End Do if Deterministic
    * If stochastic scenario set
        Do If  $\eta_c(r)$  EQ Probabilistic
    * Check if choice probability is above random probability
        Select c If  $\mathcal{I}_I(r, c, q) \geq (\sigma + \varepsilon)$ 
    * c:S(1) is the first element selected with the above Select statement
         $I(r, c: S(1)) = 1$ 
        End do if Probabilistic
    End Do q
End Do r
Select c*
Write Disk(I, M,  $U_I$ ,  $\Phi_I$ ,  $\mathcal{I}_I$ ,  $\Upsilon$ )
End Procedure

```





## Appendix 11: Behavior

Behavior is discrete and continuous. The Evaluation & Selection procedure selects the discrete choice, the behavior procedure executes it and determines the intensity of the behavior. One behavior can trigger other behavioral sequences in a tiering process. The behavior can become the stimuli to the same individual. If other stimuli (the environment) change in response to the choice (behavior), then the Evaluation & Selection procedure can reinforce that choice (behavior) in comparable future conditions.

"Doing" is the a balancing of activation (excitatory) and restriction (inhibitory) pressures/potential.

When the behavior is that of a group, the amplification should be large to note that, for example, a 10% group intent may be thousands of individuals.

Behaviors can also have tiering (conditional activation) from other behaviors. Tiering increases/inhibits the intensity, not the triggering. (The theoretical basis for this approach is based on required constraints for model behavior consistent with psychology rather than with psychology itself.)

Define Parameter Block

$L_q$  Number of Choice Sets

$\alpha_i(r,c)$  Saliency of attitude on predisposition utility of choice.

$\beta_i(r,c,y)$  Saliency of Notions on utility of a choice.

$\gamma_{th}(r,c,y)$  Saliency of Positive Incongruity on utility of choice

$\gamma_{fi}(r,c,y)$  Saliency of Negative Incongruity on utility of choice

$\epsilon$  A small number

$\eta_c(r)$  Switch to decide stochastic, group, or winner-gets-all logic

$\lambda_i(r,c,q)$  Saliency of other choices on utility of current choice

$\sigma$  Random number (Uniform, 0,1)

$\Psi_I(r,c,q)$  Map of choice set to choices

End Parameter Block

Define Variable Block

$B(r,c)$  Actual Behavior

$D_{Bh}(r,c)$  Positive Incongruity for Behavior

$D_{Bi}(r,c)$  Negative Incongruity for Behavior

$D_{Ph}(r,y)$  Positive Incongruity for a Notion

$D_{Pi}(r,y)$  Negative Incongruity for a Notion

$I(r,c)$  Choice Evaluation (Intent)

$M_B(r,c)$  Marginal Probability of Behavior  
 $N_B(r,c)$  Attitude for Behavior Utility  
 $N_{Bh}(r,c)$  Attitude of Positive Incongruity for Behavior Excitation Utility?  
 $N_{Bi}(r,c)$  Attitude of Negative Incongruity for Behavior Inhibition Utility?  
 $N_{Dh}(r,y)$  Attitude of Positive Incongruity for Behavior Utility?  
 $N_{Di}(r,y)$  Attitude for Negative Incongruity Behavior Utility?  
 $N_P(r,y)$  Attitude for Notion Utility  
 $P_E(r,y)$  Effective Notion  
 $U_B(r,c)$  Utility of the Behavior  
 $U_{Bf}(r,c)$  Utility for Behavior Excitation  
 $U_{Bg}(r,c)$  Utility for Behavior Inhibition  
 $U_V(r,c)$  Utility of Behavior for Prioritization  
 $V(r,c)$  Indicated Behavior  
 $\Phi_B(r,c)$  Weighted prioritization utility  
 $\Omega_B(r)$  Sum of weighted prioritization utility  
 End Variable Block

#### Define Parameter Block

$\alpha_V(r,c)$  Saliency of behavior from Attitude  
 $\beta_V(r,n,y)$  Saliency of behavior from existing notions  
 $\gamma_{Bh}(r,c)$  Excitation of behavior from positive incongruity  
 $\gamma_{Bi}(r,c)$  Excitation of behavior from negative incongruity  
 $\gamma_{Vh}(r,c,y)$  Saliency of behavior from positive incongruity  
 $\gamma_{Vi}(r,c,y)$  Saliency of behavior from negative incongruity  
 $\delta_{Bh}(r,c)$  Inhibition of behavior from positive incongruity  
 $\delta_{Bi}(r,c)$  Inhibition of behavior from negative incongruity  
 $\delta_V(e,c,q)$  Saliency of other behavior on subject behavior  
 $\mu_V(r,c)$  Behavior weighting for prioritization  
 $L_q(r)$  Maximum number of choice sets  
 End Parameter Block

Default  $\mu_V=2.0$ ,  $L_q=1$ ,

$\alpha_V, \beta_V, \gamma_{Bh}, \gamma_{Bi}, \gamma_{Vh}, \gamma_{Vi}, \delta_{Bh}, \delta_{Bi}, \delta_V=0.0$

#### Define Procedure Behavior

\*

Select Moment (Prior)  
 Read Disk(V)  
 Select Moment (Current)

\* Behavioral intensity

$$U_B(r, c) = \alpha_V(r, c) \times N_B(r, c) + \sum_y \beta_V(r, n, y) \times N_p(r, y) \times P_E(r, y) + \\ \sum_y \gamma_{Vh}(r, c, y) \times N_{Dh}(r, y) \times D_{ph}(r, y) + \sum_y \gamma_{Vi}(r, c, y) \times N_{Di}(r, y) \times D_{pi}(r, y) \\ + \sum_q \delta_V(e, c, q) \times N_B(r, q) \times V(r, q)$$

\* Utility of Acting is one component of incongruity and Cognitive Resource's effect on behavioral triggering.

$$U_{Bf}(r, c) = \gamma_{Bh}(r, c) \times N_{Bh}(r, c) \times D_{Bh}(r, c) + \gamma_{Bi}(r, c) \times N_{Bi}(r, c) \times D_{Bi}(r, c)$$

\* Utility of Avoidance is the other component of incongruity and Cognitive Resource's effect on behavioral triggering.

$$U_{Bg}(r, c) = \delta_{Bh}(r, c) \times N_{Bh}(r, c) \times D_{Bh}(r, c) + \delta_{Bi}(r, c) \times N_{Bi}(r, c) \times D_{Bi}(r, c)$$

\*Implied behavior

\* Behavior is the acting on a planned intent. The triggering and intensity terms (mathematically) should be combined, but are separated for conceptual clarity.

$$V(r, c) = I(r, c) \times \frac{1}{1 + e^{U_{Bf}(r, c)}} \times \frac{1}{1 + e^{U_{Bg}(r, c)}} \times e^{U_B(r, c)}$$

\*One behavior may determine the viability or potential use of another behavior in a Boolean (on –off) fashion.

$$U_V(r, c) = \mu_V(r, c) * V(r, c)$$

\* limit behavior to maximum energy load (maximum energy entity can apply to behavior)

$$\Phi_B(r, c) = e^{(U_V(r, c))}$$

$$\Omega_B(r) = \sum_c \Phi_B(r, c)$$

$$M_B(r, c) = \Phi_B(r, c) / \Omega_B(r)$$

$$B(r, c) = \min (L_q(r) \times M_B(r, c), V(r, c))$$

Write Disk (B, V, M<sub>B</sub>, Φ<sub>B</sub>, U<sub>V</sub>, U<sub>B</sub>, U<sub>Bf</sub>, U<sub>Bg</sub>)

**End Procedure**



## Appendix 12: Action

Action is just the delay of the behavior – over the time it takes to transform a behavior into a consequential action

Define Variable Block

B(r,c) I Behavior

K(r,c) Realized Action

Q(r,c) Behavior in Process of becoming Action

End Variable Block

Define Parameter Block

$\eta_K(r,c)$  Boolean for discreet or continuous action realization

$\tau_K(r,c)$  Delay time from behavior to realized action

End Parameter Block

Default  $\eta_K=0$  (continuous),  $\tau_K=6.0$

Initialize Q in equilibrium ( $Q=B$ ) or at zero. Use equilibrium during model development.

Ultimately: Initial  $Q=Q_0$

### Define Procedure Action

Select Moment (Prior)

Read Disk(Q, B)

Select Moment (Current)

\* Action is the execution of behavior – which takes time

\* B is brought in over all time. It needs to use initial value or 0.0 if  $t-\tau_K$  is less than 1.

\* The action is the continuous (Q) with time constant  $\tau_K$ ,

\* or it is the discrete delay of B delayed  $\tau_K$  “moments”, depending on the Boolean  $\eta$

$$K(r, c) = \frac{Q(e, c)}{\tau_K(r, c)} \times (1 - \eta_K(r, c)) + B(r, c, t - \tau_K) \times \eta_K(r, c)$$

\* Q is a SD material delay of B when K is continuous

$$Q(r, c) = Q(r, c) + dt \times (B(r, c, t) - K(r, c))$$

Write Disk(Q,K)

End Procedure



## Appendix 13: Stimuli

Stimuli just map the action from the behaviors of entities or the physical world to the other entities – including back to themselves. This section represents signal dissemination. Interventions are exogenous stimuli.

The estimate of the notion equation parameters will result in a statistical  $r^2$  (r-squared) that indicates the fraction of the results that are explained by the equation. The remaining unexplained part is noise that does affect the stochastic aspect of the model results. If the standard deviation between the equation results and the raw data is “d,” then a “exogenous noise stimuli corresponding to a random variable with a mean 1.0 and a standard deviation of “d” can capture this variation. The exponent for the notion formation ( $\beta_s$ ) is simply  $1.0-r^2$

Define Variable Block

K(r,c) Action

S(r,z) Stimuli

End Variable Block

Define Parameter Block

J(r,d,c,z) Transfer Matrix (0.0=Default; all values are 0.0 or 1.0)

$\varepsilon$  a small number

X(z,c) Exogenous Intervention

End Parameter Block

Default J=0.0; all values are 0.0 or 1.0)

### Define Procedure Stimuli

\* Map Action to stimuli across social network

$$S(r, z) = \text{xmax}(\varepsilon, J(r, d, c, z) * \text{xmax}(K(d, c), X(d, c)))$$

Write Disk (S)

End Procedure





## Appendix 14: External Conditions

The code below is conceptual and illustrative. The definition in an implemented model would depend on the specific issues and interventions the model was meant to address. The Actions (K) come into the sector which then creates new actions (K) that become stimuli for the society entities. Care must be taken that the physical model contain societal choices -- such as the choices to sell goods in a store. There can be no redundancy between the "explicit" decisions in the cognitive procedures and those "implicit" in (external) physical procedures.

In the example below, the “ $\alpha$ ” are estimate constants. All growth rates (Gr) are scenario parameters.

As an example, data could to build the external procedure could come from: International Futures Model database, WDI (World Bank), GTAP Model database

Per the Cobb-Douglas formulation, the sum of the parametric exponents on GDP sum to unity – for example, if one added materials or energy explicitly.

In this example, Fertility is exogenous and backed out, knowing net growth and death rate from data, PopLifeTime is exogenous from data, and CapLifeTime=20 years (GTAP data).

### Define Procedure External

\*

Select MOMENT (PRIOR)

Read Disk(K,Pop,Capital,A,IndicatedResRevenue)

Select MOMENT (CURRENT)

\*

\* Population logic

$BirthRate = Pop * Fertility$

$DeathRate = \frac{Pop}{PopLifeTime} - \alpha_{Murder} \times \sum_e K(e, murder)$

$Pop = Pop + dt \times (BirthRate - DeathRate)$

\*

\* Natural Resource Revenue

\* Indicated revenue reduced by destruction that are mitigated by policing

*IndicatedResRevenue*

$$= \text{IndicateResRevenue} + dt(\text{IndicateResRevenue} \times \text{ResRevGr}) \\ - \alpha_{\text{Distruction}} \times \sum_e K(e, \text{Distruction}) \times e^{-\alpha_{\text{Policing}} \sum_e K(US, \text{Policing})}$$

\* Resource Revenue reduced by disabling activities that are mitigated by policing  
*ResRevenue* =

$$\text{IndicatedResRev} - \alpha_{\text{Disabling}} \times \sum_e K(e, \text{Disabling}) \times e^{-\alpha_{\text{Policing}} \sum_e K(US, \text{Policing})}$$

$$\text{PopResRevenue} = \text{ResRevenue} * \alpha_{\text{PopResRev}} \times K(\text{gov}, \text{PopresRev})$$

$$\text{GovResRev} = \text{ResRevenue} * \alpha_{\text{GovtResRev}} \times K(\text{gov}, \text{GovtResRev})$$

\*

\*Drug Revenue is exogenous

$$\text{GovDrugRev}(e) = \text{DrugRev}(e) \times \alpha_{\text{GovDrugRev}}(e) \times K(e, \text{DrugRev})$$

$$\text{PopDrugRev} = \sum_e \text{DrugRev}(e) \times \alpha_{\text{PoPDDrugRev}}(e) \times K(e, \text{DrugRev})$$

\*

$$\text{TaxRate} = \alpha_{\text{TaxRate}} \times K(\text{gov}, \text{TaxAction})$$

$$\text{GovRevenue} = \text{GovResRev} + \sum_e \text{GovDrugRev}(e) + \text{TaxRate} * \text{GDP}$$

\*

\* Government Use of Funds

$$\text{GovCorruption} = \text{GovRevenue} \times \alpha_{\text{GovCorruption}} \times K(\text{gov}, \text{Corruption})$$

$$\text{PopGovSpending} = \text{GovRevenue} \times \alpha_{\text{GovCivilSpending}} \times K(\text{gov}, \text{Spending})$$

$$\text{GovInvest} = \text{GovRevenue} \times \alpha_{\text{GovInvest}} \times K(\text{gov}, \text{Invest})$$

\* Cost of Gov operations is exogenous

$$\text{GovDebt} = \text{GovRevenue} - \text{PopGovSpending} - \text{GovInvest} - \text{GovCorruption} - \\ \text{InterventionCost} - \text{GovOperations}$$

\*

\*Indigenous Economy

\* Disruption of Material flows or factory operations

$$\text{Disruption} = e^{-\alpha_{\text{Disruption}} \sum_e K(e, \text{Disruption})}$$

\* Intimidation of Labor

$$\text{Insecurity} = e^{-\alpha_{\text{Insecurity}} \sum_e K(e, \text{Violence})}$$

\* Investment

$$\text{Invest} = \text{Invfraction} * \text{GDP} * \exp(\alpha_{\text{Security}} \times K(e, \text{Crime}) + K(US, \text{Invest}) + \text{GovtInv})$$

$$\text{CapRetirement} = \text{Cap}/\text{CapLifeTime}$$

\* Capital Stock

$$\text{Capital} = \text{Capital} + dt \times (\text{Invest} - \text{Retirement})$$

\* Technology/Education GrowthRate (AGr) is exogenous

$$A = A + dt \times (A \times \text{AGr})$$

\*Capital productivity loss form Infrastructure Functionality due to security (materials and operational availability); Labor intensity loss from security. Assumes constant fraction of Population as Labor.

\* Gross Domestic Product

$$GDP = GDP0 * A * \left( \frac{Capital}{Capital0} * Disruption \right)^\alpha * \left( \frac{Population}{Population0} * Insecurity \right)^\beta$$

\* Per Capita Income

$$CapitaGDP = (DGP + PopDrugRev + PopResRev)/Pop$$

\*

\* Resistance and Violence

$$Resistance = \alpha_{Resistance} \times \sum_e K(e, Resistance)$$

$$Violence = (USMilitaryPresence + GovMilitaryPresence) * \alpha_{Violence} \times Resistance$$

$$ResistanceCost = (USMilitaryPresence + GovMilPresence) * \alpha_{ResistanceCost} \times Resistance$$

$$InterventionCost = (USMilitaryPresence + GovMilPresence) \times \alpha_{InterventionCost} \times Resistance$$

\*

\* Consequences to Stimuli

$$K(Economy, WellBeing) = (CapitaGDP/CapitalGDP_0)$$

$$K(Economy, Crime) = \beta_{crime} \times \sum_e K(e, violence)$$

$$K(e, GovtInfluence) = \alpha_{Influence} \times DrugRev(e)$$

$$K(e, ResistanceCost) = ResistanceCost$$

\*

Write Disk(K,Pop,Capital,A, IndicatedResRevenue)

**End Procedure**



## Appendix 15: Limitations of a Fixed Blueprint

To use the model, we always start (via the client) to determine the behaviors of interest (BOI). From there, with SMEs, we hypothesize all the stimuli that might affect those behaviors. With data, we can then corroborate or falsify the hypothesis. The statistical process produces model parameterizations along the way. The biggest, unavoidable, uncertainty in the model is to not have all the key behaviors and to not have recognized the associated stimuli. Given the stimuli and behaviors, we can then work with the SMEs to extract the Cognitive Resources.

The Psychological Engine contains a fixed “blueprint” of how cues follow the paths to specific choice selections. The potential choices and their relevant cues typically come from (SMEs) or are implied from the reviews of government documents and news media pertaining to an individual. Historical data can directly allow the testing of hypothetical choice paths and the parameterization of choice equations. Several researchers have suggested a means to explore for new response options, thereby having a more human-like, fluid choice set. Mugan and Kuipers (2007, 2008, 2009a, 2009b) consider learning from experience for development of action acquisition. They start with physical constraints and let a computerized agent learn how to perform tasks from all the movement options available to the (robotic) agent. Jensen et. al. (2008) consider non-experimental (historical) data to determine the casual (behavioral) relationships involved. Still and Crutchfield (2007a, 2007b) look for the minimum causal complexity that explains relationships in time-series data sets. The minimum information approach would be consistent with the evolution of behavior and the limitations of human cognitive capacity to process information in real-time (Gigerenzer and Goldstein 1996). Lastly, Glymor et. al. (1999,2000) propose methods to causal discovery in data sets by testing the multiple causal paths simultaneously and selecting those that are most likely and without contradictions. In all cases, such methods could be applied to enhance/modify or help validate the fixed-relationships of the blueprint embodied in a psychological engine. Further, the use of such methods are only viable when sufficient data exists for past choices. None have the ability to invent new solutions (choices) to new conditions (problems). Therefore, although these methods are an adjunct to the psychological approach used in the SNL psychological engine, the fixed blueprint provides the best available means to simulate human response to varying cues and the learning associated with those responses.

One aspect of societal behavior is the “discovery” of new leaders that may strongly affect future societal dynamics. If the modeled (homogenous) populations are first divided into arbitrary (or rationalized) factions containing variation capturing the actual heterogeneity of populations, then the psychological model can obtain characteristics of agent-based models. If representative individual are then arbitrarily fragmented from these “heterogeneous” populations, but are well connected to them (via the signal dissemination), then the model could produce dynamics that represent the further fragmentation of society and changing roles of behavior. Initially a potential

or nascent leader is “hidden” in the noise of the “associated” population. Given herding behaviors and internal feedback within a societal group, exogenous actions can affect both the individual and the potential leaders with the dominance of the potential leader reinforcing herding (i.e., leadership dynamics). These responses can cause a bifurcation in overall societal behaviors and cause the leader in grouping to actually appear as such.

## Appendix 16: Estimation and Parameterization

### Estimation and data collection

The primal unit-of measure for the model is “stimuli units per unit of time.” They are actually an index based on a numeraire. Because there is no absolute behavioral referencing in the real world, we have to think of all variables as primarily ordinal and that they must conform to a purely affine mathematical theory. Just as physical-law equations are independent of the units of measure, so must the cognitive model. The numeraire is an agreed upon and consistently used value that enables the ranking of variables within the same contextual meaning.

For example, notion is a pattern of stimuli (a collection). The magnitude of the pattern is a weighted-function of the stimuli:

$$P = K * \prod S^{\alpha}$$

Or, as an linear equation whose parameters can be estimated with historical data:

$$\ln(P) = \ln(K) + \sum a * \ln(S)$$

where K is a scaling constant and  $\alpha$  are weights. The weights need only be relative. That is, if S(2) is twice as important as S(1), then  $\alpha(1)$  can arbitrarily be set to any value (e.g. unity) and  $\alpha(2)$  is twice that value (e.g., 2). Any common scaling issue among the  $\alpha$  are mathematically cancelled because all modeled mechanisms will be comparing (numerator over denominator) like terms.

The two equations above have unit problems in that the exponent affects the implied unit of measure (i.e., as a physical example, feet –squared are an area in square feet and feet-cubed are a volume in cubic-feet-- both being different from the simple measure of length in feet. ). The “real” equation for perception is:

$$\frac{P}{P_0} = \prod \left(\frac{S}{S_0}\right)^{\alpha}$$

Or

$$P = P_0 * \prod \left(\frac{S}{S_0}\right)^{\alpha}$$

where the subscripted variable is the numeraire value. For the initial modeling, the variables will be used as if they are an index, that is, we treat them a normalized value (an “actual” divided by an arbitrary numeraire) of, for example, stimuli.

The model theory assumes a choice is always in the context of multiple options.

As a simple example, the probability ( ) of selecting a choice “i” using a single Notion (P) is:

$$\mathcal{P}_i = e^{a_i + \beta_i * P} / \sum_j e^{a_j + \beta_j * P}$$

If there are only two choices in the choice set and one is a numeraire, the  $\alpha$  and  $\beta$  for it are zero such that:

$$\mathcal{P}_i = e^{a_i + \beta_i * P} / (1 + e^{a_i + \beta_i * P})$$

Or

$$\mathcal{P}_i = 1 / (1 + e^{-(a_i + \beta_i * P)})$$

(See Ben-Akiva 1985)

If the Notion (perception) is assumed to be identical to the measureable stimuli (S) and the probability of the choice is assumed to equal the frequency of the measured behavior, then the  $\alpha$  and  $\beta$  can be estimated by least-squares using the equation:

$$\ln\left(\frac{1}{\mathcal{P}_i} - 1\right) = a_i + \beta_i * S$$

The information for the regression can come from textual news items, subject matter experts (SME), or synthetic data generation using hypothesized scenarios (stimuli) with estimated responses from SMEs. Parameters are refineable (validated) by “pinging” the actual subject of interest with minor stimuli.



If there are multiple choices, the following equation can be used for least-square estimation by assuming choice “i” is the numeraire (its  $\alpha$  is 0.0 and  $\beta_i = \beta_j$  for a single S):

$$\ln\left(\frac{\mathcal{P}_j}{\mathcal{P}_i}\right) = a_j + \beta_j * (S_j - S_i)$$

Least square estimation is biased and maximum likelihood estimation is preferred, but more complicated (Ben-Akiva 1985). Least Square estimation is adequate for scoping studies. The estimation process becomes marginally more complex with additional independent variables, but only in terms of distinguishing the parameters to be estimated.

The estimate process needs to parameterize and calibrate each component of the engine, from stimuli through to behavior. Limited data can estimate all parameters except those with long-term time-dependency, such as cognitive resource conditioning. Much longer time-series data is needed to parameterize conditioning responses, although SME information could provide a working estimate of the parameterization. The model can run in a short-term” mode (days to weeks) that is relevant to most influence operations without the need for simulating conditioning. Longer-term runs (weeks to years) do not necessarily require conditioning simulation when the changes are within the bounds of historical conditions. When intervention do imply radically different circumstances for entities, leaning (conditioning) is probably relevant, and if not directly parameterized, needs to be part of sensitivity analysis. This section concerns its self with the routine parameterization of the engine when conditioning is not a limiting consideration.

If we have a "population" or group of individuals as an entity, there is an assumption of homogeneity. Sensitivity analysis can determine whether there is a need to divide up the population into more distinct groupings – as data support such a distinction.

The choice utility is often composed of explicit factor ( $b*x$ ) and unknown (assumed constant) factors (constant  $a$ ) such that the Utility of a choice "j" could be  $U(j)=a(j)+\text{sum}(b*x)$  Where  $b$  and  $x$  are the vectors of the parameters (weights) and the input information respectively. If the utility takes the form of the Weber-Fechner law (Weber 1978, Fechner 1907), the choice equation 1 in Chapter 4, reduces to the notion and attitude equations in Appendices 4 and 7 respectively. The actual use of attitudes in utility equations corresponds to the Steven’s law (Stevens 1957, 1961, 1975). The utility equations in the model generally assume a linearization of Steven’s law consistent with utility-function development (Keeney & Raiffa 1976).

### Stimuli and Notion Parameters:

Stimuli parameters can potentially be estimated via actual data, but will probably be based on SME input. In the absence physical of data, the notion can be simply made equal to a single stimuli. Solve for P (the notion of appendix 7) using the log of equation and Taylor series expansion to linearize " $R^a$ " (attitude) terms (if cognitive resources " $R$ " are not assumed constant.) Notions are immediate or delayed stimuli. The time constants can be directly estimated (Hamilton 1980).

### Expectations:

Expectations are primarily delayed perceptions. Surveys, SME's and synthetic data can provide an estimate of the delay time by simply comparing the subjects stated current or "normal" value of a stimuli to its actual time series. Any direct filtering of stimulus and notion is adequately portrayed as simply a constant ratio, the delay is readable off Figure 1 below for a first order delay response by comparing the perceived current value to the stated normal value. Expectations require a double sequence of delays at a minimum. The delay time estimate is only sensitive to the total delay time for the sum of delays. The break down across the delays only affects the response at the third decimal place. With sufficient data direct estimation is possible (Hamilton 1980). The first delay (see Appendix 8), removes the noise for the data (see Figure 2); the second delay is the remembered condition.

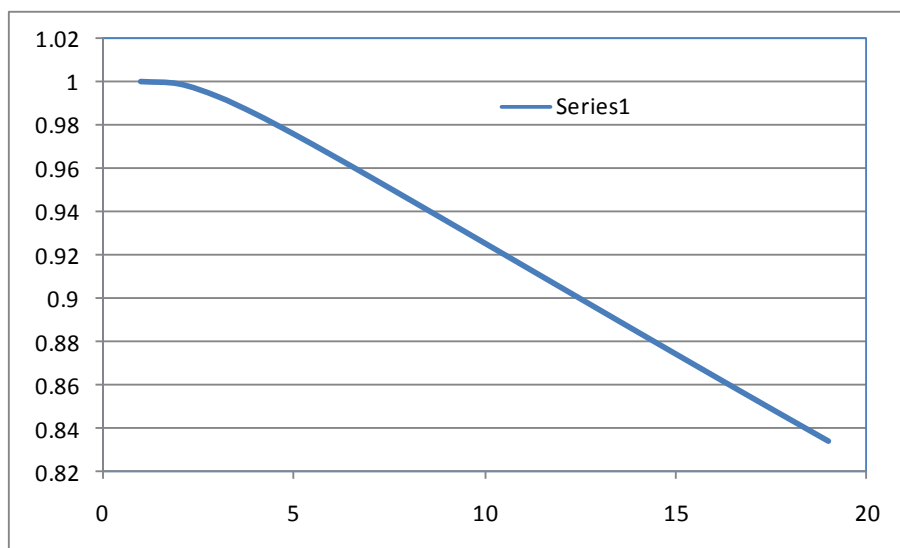


Figure 1: Ratio of immediate to delay response with delay time

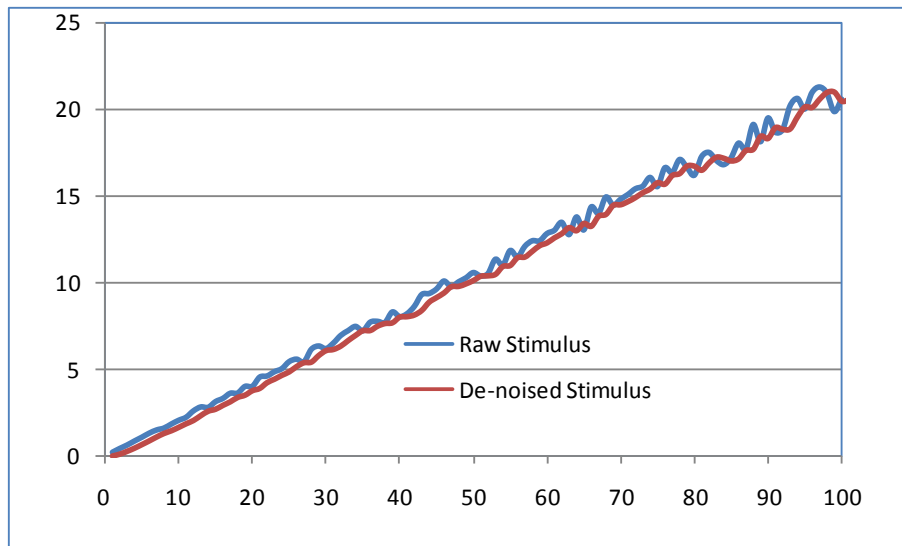


Figure 2: Impact of de-noising

The minimum delay time must be at least two time units (“moments” in the model) to extract information from the time series this limits is the classical Nyquist-Shannon Sampling Rate (Nyquist 2002, Shannon 1998).

### **Incongruity and Passivity Parameters:**

As a default we assume an offset of 10% (see Appendices 5 and 6). Experimental data may be the most useful, but survey would be most useful; SME’s may be an adequate substitute. Extended times-series data is needed for passivity estimation. (Passivity is a very secondary behavior that can usually be neglected without loss of accuracy.)

### **Behavior Parameters:**

The core estimation process actually occurs at the Evaluation and Selection procedure (See Appendix 10) of the model. Because the estimation is performed with actual data through the assumption that behaviors directly stems from Selection (Intent), we only need to know the discrete choice and not the intensity. With the parameterization of intent, the estimate can then branch forward and backward to the notion, expectation, stimuli estimations, and to the behavior parameterization, respectively.

The behavior is the same as Selection but the estimated Selection is now the assumed input and compared to actual (or synthetic) data correlating choice selection to behavioral intensity.

### **Cognitive Resources:**

This parameterization is the most complicated requiring multi-step statistical processes and linearization. When it is deemed inactive, the model will automatically (in a theoretically valid sense) compensate if the cognitive resources are assumed to have an arbitrary constant value. (The model uses affine mathematics, such that variable values only have a relativistic meaning in context. There may be existing long-term studies that can “help” characterize parameters (time constant and gain) of cognitive resource conditioning and atrophication (see Appendix 9). A more comprehensive approach would be to simultaneously estimate all the parameters of the entire model over time using a Kalman Filter (Kalman 1960).

### **Calibration:**

Estimation produces the best estimate (with uncertainty) for model parameters. Because the model is a finite abstraction of reality, it necessarily omits aspects of the actual system that are not relevant to the “problem of interest.” Therefore, for the model to exactly reproduce historical values and to correct for biases, parameters may require small adjustments – called calibration. The feedback loops in the model must exactly balance in equilibrium. If we have time series data, all state variable and their derivative must be consistent. The estimation process definitionally generates an error term ( $\varepsilon$ ) that for time series will be a function time -  $\varepsilon(t)$ . To calibrate, the constants  $\alpha$  are modified to :

$$\alpha(t) = \alpha + \varepsilon(t)$$

If the  $\varepsilon$  drives  $\alpha$  outside of its bounds of standard error, it indicates a faulty estimation process or missing mechanism in the model.

### **Some Pedagogic Examples:**

This example is to show the use of data and solution logic for a simple voter-choice model. It is solved algebraically to demonstrate the concept. With more data and the recognition of uncertainty, we use a regression process to estimate parameters. In this example “M” is the voter share among three candidates. The information voters use to evaluate candidates is assumed to be only corruption (C) and Threats (T). The voter notion of C and T are obtained by surveys or possibly the review of news media. The data are shown in Table 1.

		Survey Score					
"j" candidates		Corruption		Threats		Vote share	
Candidate 1	J1	9	C1	2	T1	0.5	M1
Candidate 2	J1	6	C2	4	T2	0.3	M2
Candidate 3	J3	3	C3	8	T3	0.2	M3
		Measured Perception Input				Measured Behavioral Output	

Table 1: Voter Data

Per the logic of Qualitative Choice Theory:

$$Utility(j)=U_j=a(C_j)+b*(T_j) \quad \text{Equation A.1}$$

$$M_j=\exp(U_j)/\sum(i)(\exp(U_i)) \quad \text{Equation A.2}$$

$$M_j/M_i=\exp(U_j)/\exp(U_i) \quad \text{Equation A.3}$$

$$\ln(M_j/M_i)=U_j-U_i=a*(c_j-C_i)+b(T_j-T_i) \quad \text{Equation A.4}$$

Equation A.4 is the linear equation for which conventional regression methods can estimate parameters. There are only two equations and two unknowns because  $1-M_1-M_2=M_3$ . The third equation (for  $M_3$ ) is residual value. Table 2 shows the values for estimating the Equation A.4.

Share Term	Value		Delta C	Value		Delta T	Value
$\ln(M_2/M_1)$	-0.51083		$C_2-C_1$	-3		$T_2-T_1$	2
$\ln(M_3/M_1)$	-0.91629		$C_3-C_1$	-6		$T_3-T_1$	6

Table 2: Equation Estimation

Via simple substitution the estimated parameters are:

$$a= 0.20539538$$

$$b= 0.052680258$$

Reinserting these parameter in the actual model Equations A.1 and A.2, Table 3 show that the parameters do indeed produce the expected outcome

Test Param		Estimate	$\exp(U_i)$
m1		0.5	7.056286587
m2		0.3	4.233771952
m3		0.2	2.822514635
sum		1	14.11257317

Table 3: Cross-check of results

The next example combines all the components of the engine in a illustrative example considering the escape from a building fire. Table 4 shows the values of the base case stimuli and Table 5 shows the value of Cognitive Resources. The entity is in a room on the second floor with a fire

spreading at a moderate rate. There is a fire extinguisher and the entity is somewhat more oriented toward heroism than survival as the driving consideration. As shown in Figure 6, the primary behaviors (and Selection) are Fight to Flight. If the choice is to flee, there are three options: use the stairwell, window or roof to seek escape. In the base case, the choice is to fight with an intensity of 5.13 and shown in Table 6..

Stimuli		Stimulus Intensity
S1	Fire	1
S2	Fire Spread Rate	2
S3	Fire Proximity to Stairwell	1
S4	Fire Extinguisher	1
S5	Stairs Down	1
S6	Window	1
S7	Room Elevation	2
S8	Ladder to Roof	1
S9	Proximity of Fire Station	1

Table 4: Basecase Values of Stimuli.

Cognitive Resource		Value
R1	Heroism	2
R2	Survival	1

Table 5: Basecase Value of Cognitive resources

Available Behaviors	Executed Behavior
-Fight or Flight Behavior Set	
Flight	0.00
Fight	5.13
-Exit Behavior Set	
Stairwell	0.00
Window	0.00
Roof	0.00

Table 6: Basecase Behavior

If the Cognitive Resource of heroism is reduced to unity there is low intensity flight to the roof as shown in Table 7.. This occurs because there stimuli that the fire is slowly growing, it is near the steps down, and there is fire station close by (it can come well before the fire spreads to the roof.)

Available Behaviors	Executed Behavior
-Fight or Flight Behavior Set	
Flight	0.35
Fight	0.00
-Exit Behavior Set	
Stairwell	0.00
Window	0.00
Roof	0.41

Table 7: Heroism reduced to unity

The simulated entity is afraid of heights, If the fire were on the first floor, than the entity with reduced “heroism” would escape via the window as shown in Table 8.

Available Behaviors	Executed Behavior
-Fight or Flight Behavior Set	
Flight	0.35
Fight	0.00
-Exit Behavior Set	
Stairwell	0.00
Window	0.41
Roof	0.00

Figure 8: Heroism reduced to 1 and on first floor

If the entity is still “heroic,” but there is no fire extinguisher, and fire is not near the stairwell, then the entity will use the stairwell as shown in Table 9.

Available Behaviors	Executed Behavior
-Fight or Flight Behavior Set	
Flight	0.35
Fight	0.00
-Exit Behavior Set	
Stairwell	0.41
Window	0.00
Roof	0.00

Table 9: Fire not near stair and no fire-extinguisher

Lastly, in Figure 10, the stairs is open and the fire extinguisher is available. In this instance the entity will still fight, but because of escape options, not with the same vigor as in the case where escape is also risky.

Available Behaviors	Executed Behavior
-Fight or Flight Behavior Set	
Flight	0.00
Fight	0.35
-Exit Behavior Set	
Stairwell	0.00
Window	0.00
Roof	0.00

Figure 10: Heroism reduced to 1, but stairs and fire-extinguisher available.

The above is a static example. If the action of the entity changes the condition of the fire, or the fire dynamics change significantly, there would be a dynamic set of responses. For example, if basecase response (Fight) does not reduce the growth of the fire, the stairwell becomes less accessible than the entity is forced to the roof.

The generalization of the process allows any behaviors for any number of entities, having any number of choices and choice sets to be realistically modeled.



## Appendix 17: From Blueprint to Parameterized Model

The Blueprint is a vehicle (currently embodied in an EXCEL spreadsheet for subject-matter experts - SME's - to fill in) that describes the relationships that lead from Cues through Behavior for entities of interest. The blueprint defines the configuration of the model. The model allows a Bayesian information fusion of SME, formal data collections (time-series history), surveys (with de-biasing), experimental (psychological) results, pinged-responses of the actual system, and news media/anecdotes for parameterization. The varied information flows can act as constraints on the formal parameter-estimation process or can be used to hierarchically specify parameterization priorities. In general, the SME input will act as the primary means to initiate a modeling effort. The use of uncertainty quantification can then determine the importance of, and thus the need for, additional data. Generic values (based on analogous conditions available in historical data) can act as the “Bayesian” priors in the absence of specific corroborating information.

This discussion explains the mapping from blueprint to the computational model.

### 1. Parts of Blueprint

Each entity in the model is based on the “Blueprint” representation of how that entity responds to cues (stimuli). Although the representation hinges on the behaviors, the blueprint is meant to capture, the logical (causal) flow begins with the cues. Figure 1 notes examples of potential cues.

CUE CATEGORIES		CUES (RELEVANT STIMULI)	
1	<b>Foodstuff availability and gov. popularity</b>		
		C1	Domestic food price index relative to general price index (FPI/PI)
		C2	Domestic crop productivity
		C3	Foodstuff accessibility
2	<b>Population stability and gov popularity</b>		
		C4	Low SES voter trend sentiment (LVS)
		C5	Med SES voter trend sentiment (FUTURE)
		C6	Degree of employment for low SES
		C7	Degree of employment for Med SES
		C8	Low SES protests against gov (LVP)
		C9	Med SES protests against gov (FUTURE)
3	<b>revenue</b>		
		C10	Externally generated gov revenue
		C11	Internally generated gov revenue

Figure 1: Example Cues

The cues activate low-level beliefs that are the equivalent of simple perceptions or notions. Figure 2 notes examples of potential Beliefs and the Cues that activate them. The numbers in parentheses are the relative importance of each cue to the belief.

EXTERNAL (FEATURE) BELIEF		CUE ACTIVATION	
		Low Value	High Value
B1	Common foodstuff is less available	C3(50)	C1(50);
B2	Common foodstuff is more available		
B3	Increase in Low SES support		C4(100)
B4	Decrease in Low SES support	C4(100)	
B5	Increase in Med SES support		
B6	Decrease in Med SES support		
B7	Ability to fund gov programs		C10(80), C11(20)
B8	Inability to fund gov programs	C10(80), C11(20)	
B9	Gov revenue less than gov expenses	C10(80), C11(20)	

Figure 2: Example Beliefs

Figure 3 notes how beliefs prime motivators (M#). Motivators are the high-level concerns the beliefs invoked as note in Figure 4. Figure 3 also show shows that beliefs have emotive (affectual) components (AFF#). These affects can be negative or positive and their weights are shown in Figure 5.

BELIEF	BELIEF OUTPUT PRIMES	General Affect associated with Activated Belief
B1	M1, M3	AFF1
B2	M5	AFF16
B3	M1	
B4	M1, M3, M5, M6	
B5	M5	
B6	M1, M4, M5	
B7	M3, M5., M6	
B8	M1, M2	
B9	M2	

Figure 3: Example Priming

POTENTIAL BEHAVIOR MOTIVATORS		MOTIVATOR ACTIVATION	MOTIVATION OUTPUT PRIMES
M1	Minimize perception of gov failure	ATT+PSN+PBC	B12
M2	Increase gov revenue	ATT+PSN+PBC	B16
M3	Increase availability of food	ATT+PSN+PBC	
M4	Increase popularity of gov		
M5	Maximize perception of gov effectiveness		
M6	Increase gov services		B16

Figure 4: Example Motivations

	AFFECT ASSOCIATED WITH BELIEFS (AFF)	
	Negative	Positive
B1	Neg = 8	Pos = 0
B2	Neg = 0	Pos = 8
B3	Neg = 0	Pos = 6
B4	Neg = 10	Pos = 0
B5	Neg = 0	Pos = 8
B6	Neg = 7	Pos = 0
B7	Neg = 0	Pos = 8
B8	Neg = 10	Pos = 0
B9	Neg = 8	Pos = 1

Figure 5: Strength of Belief Affect

The strength of the motivators is determined by the Attitudes (Object, Norm, and Control) as shown in Figure 6. Figure 7 indicates the strength of those attitudes.

Motivators	Object Attitude toward Motivating Behavior	Perceived Social Norm towards Motivating Behavior	Behavioral Control associated with Motivating Behavior
M1	ATT1	PSN 1	PBC 1
M2	ATT10	PSN 10	PBC 10
M3	ATT11	PSN 11	PBC 11
M4			
M5			
M6			

Figure 6: Attitudes associate with Motivators

Motivator	OUTLOOK TOWARDS MOTIVATORS	VALUE	PERCEIVED SOCIAL NORMS	VALUE	PERCEIVED BEHAVIORAL CONTROL	VALUE
M1	ATT1	Pos = .30	PSN 1	0.6	PBC 1	0.4
M2	ATT1	Pos = .90	PSN 2	0.7	PBC 2	0.6
M6	ATT6	Pos = .80	PSN6	0.6	PBC6	0.5

Figure 7: Strength of Attitudes

Figure 5 above show that the motivators are associated with behavioral intentions. Figure 8 shows the potential intentions and additional cues (by the intensity) required to evoke the intent.

BEHAVIORAL INTENTIONS		CUE DETERMINANTS FOR BEHAVIORAL INTENTION SELECTION	
		Low Value	High Value
B11	Promote gov successes		C2, C3, C6, C7, C10
B12	Suppress dissent	C8, C9	C8, C9
B13	Increase external revenue	C10	
B14	Increase internal revenue	C11	
B15	Increase gov import of food	C2, C3	
B16	Increase gov support of internal food production	C2, C3	
B16	Increase gov funding of Low SES programs	C4, C6,	C9

Figure 8: Behavioral Intentions

The different intentions lead to actual behaviors, examples of which are shown in Figure 9.

	POTENTIAL BEHAVIOR (PB)
PB1	To double negative comments regarding non-gov pol organizations
PB2	To threaten/arrest/harass opposition leaders
PB3	To set a price cap on targeted foodstuff
PB4	Gov to purchase medium amounts of foodstuff from world market
PB5	Gov to purchase large amounts of foodstuff from world market
PB6	Increase in number and scope to government work programs
PB7	Gov subsidies of domestic food production
PB8	Gov purchases of imported foodstuff on open market

Figure 9: Potential Behaviors

Figure 10 shows how the intentions relate to behaviors. Depending on what beliefs determine the behaviors, their affect increases the intensity of the behaviors (see Figure 5 and Figure 3).

BELIEFS	OUTCOME OF INTENTION SELECTION		
	Positive Valence	Med. Valence	Negative Valence
B11			
B12	PB1	PB1	PB1, PB2
B13			
B14			
B15	PB8 (SMALLER)	PB8	PB8 (LARGER)
B16	PB7 (SMALLER)	PB7	PB7 (LARGER)
B16	PB6 (SMALLER)	PB6	PB6 (LARGER)

Figure 10: Relationship of Intentions to Behavior

The considerations above reflect the flow from cues to behavior when there is no learning involved. Learning is driven by the occurrence (frequency) of beliefs and the incongruity they engender relative to the learned capability to deal with that level (value) of beliefs, as shown in Figure 11.

	FREQUENCY RELEVANCE OF BELIEF STATE		
	TYPICAL FREQ	TYPICAL VALUE	INTERNAL CONGRUITY
B1	1/12	1	HIGH
B2	1/12	3.4	HIGH
B3	1/6	1	HIGH
B4	1/6	1	HIGH
B5	1/6	1	MED
B6	1/6	2	MED
B7	1/12	1	HIGH
B8	1/12	1	HIGH
B9	1	1	HIGH

Figure 11: Relevance of Beliefs to Learning

The duration of the current condition (beliefs) indicate how long the entity has had to adjust to (learn from) those beliefs. This datum is captured in the entry of Figure 12.

<b>Duration of Current Environment</b>
<b>Time units</b>
20

Figure 12: Duration of Conditions

Additionally, the occurrence of belief leads to expectations. How long ago the beliefs occurred affects the residual remembered magnitude of that experience and indicates the formation of expectations. Further, the incongruity indicates the difference between the expectation and the current magnitude of the (sensory-generated) belief. These data are shown in Figure 13:

BELIEF	RECENCY RELEVANCE OF BEHAVIOR		Belief Incongruity
	TYPICAL RECENCY	PSY MAGNITUDE	DIFF
B1	6	HIGH	HIGH
B2	6	HIGH	HIGH
B3	6	LOW	LOW
B4	6	MED	MED
B5	6	HIGH	HIGH
B6	6	HIGH	HIGH
B7	6	MED	HIGH
B8	2	MED	MED
B9	2	LOW	LOW

Figure 13: Beliefs and Expectations.

To make sense of time dependence, the concept of time must be explicit and have a specific unit of measure. The Blueprint contains an obvious statement of the time unit as noted in Figure 14.

<b><u>Month</u></b>	<b><i>Applicable Time Scale</i></b>
---------------------	-------------------------------------

Figure 14: Time Measure

## 2. Transformation

This section shows how the psychology inherent in the blueprint relates to the structure of the computation model. Each translation below maps the previous translation closer to the computationally-modeled psychology. No information or logic is conceptually lost. Information may be subsumed in new parameters but (via data) can be reestablished. Figure 15 shows the logic flow of the blueprints. The “Realizable behavior” captures the added feature of the blueprint where both Motivators and Cue (implicitly representing Beliefs) determine what Behaviors are possible.

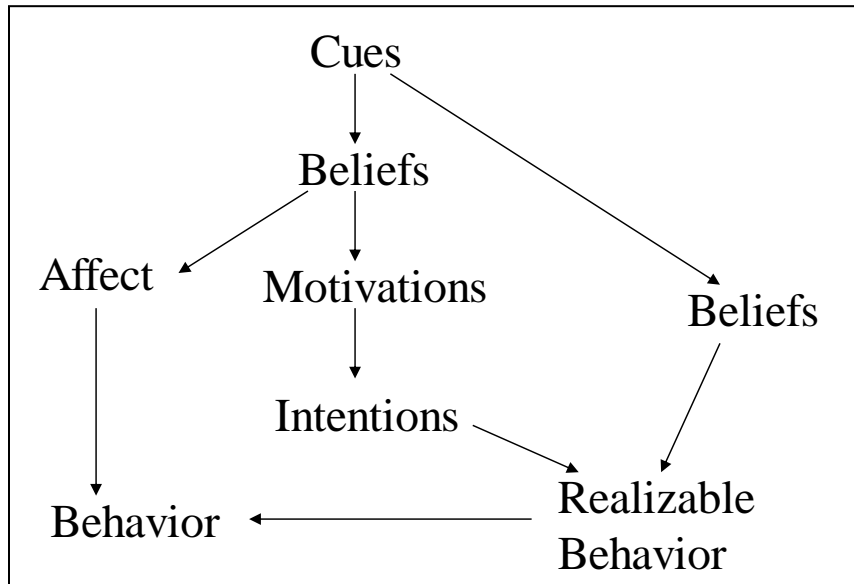


Figure 15: Blueprint Psychology

In this context, beliefs are low level perceptions such as belief that there is danger. Realizable Behaviors are actually realizable intentions in a modeled sense. Figure 16, shows this first transformation.

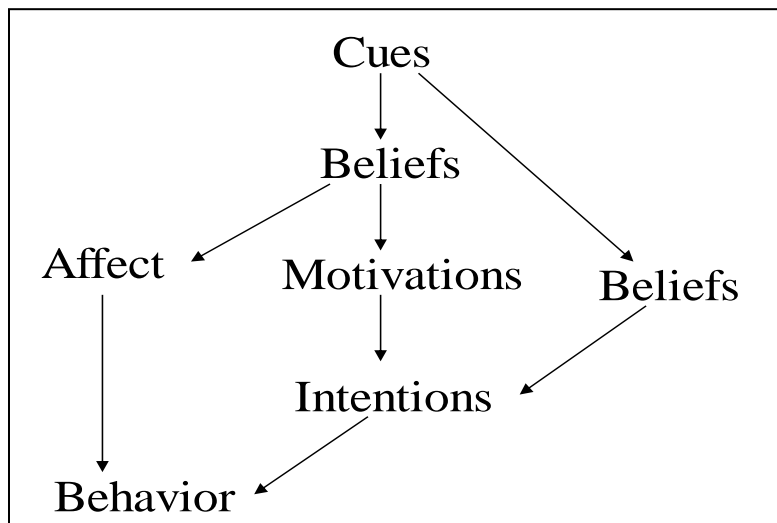


Figure 16: Transformation I

The realizable aspect of the intention and the motivation part are both reflective of the utility of the intention. The resulting intention is then the causal outcome (consequence) of the preceding utilities. This process is just a causal re-ordering of what is essentially a simultaneous process as depicted in Figure 17.

All beliefs contain an aspect of affect. This generalization means that when beliefs are generically (emotive + reasoned) specified, they directly drive psychological outcomes.

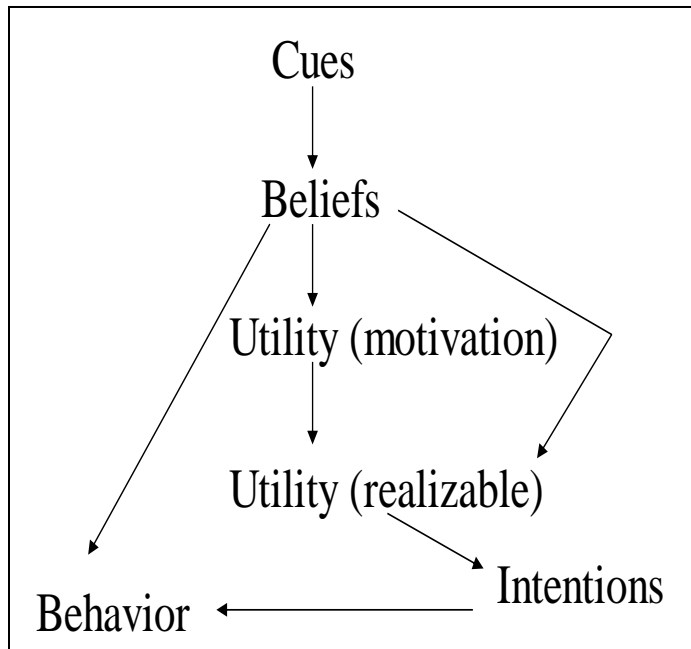


Figure 17: Transformation II

Low-level beliefs correspond to “Notions” in the computational model. The two aspects of utility may interact and are thus better quantified as a single “integrated” utility, as noted in Figure 18.

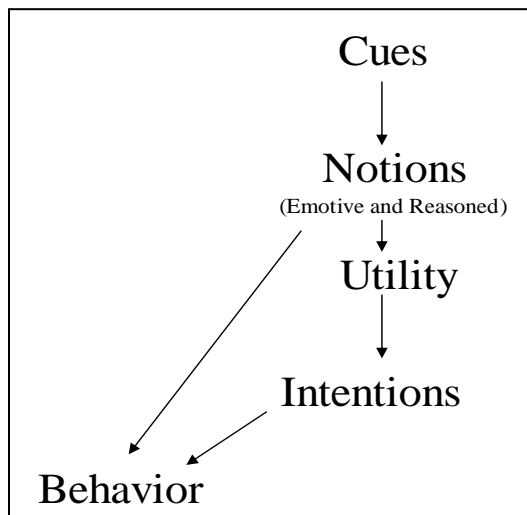


Figure 18: Transformation III

To avoid confusion related to alternative meanings of the term “intention” (in the model defined as the choice corresponding to potential behavior), the term “intention” is replaced by “Evaluation and Selection’ (and based on QCT). This transformation is shown in Figure 19.

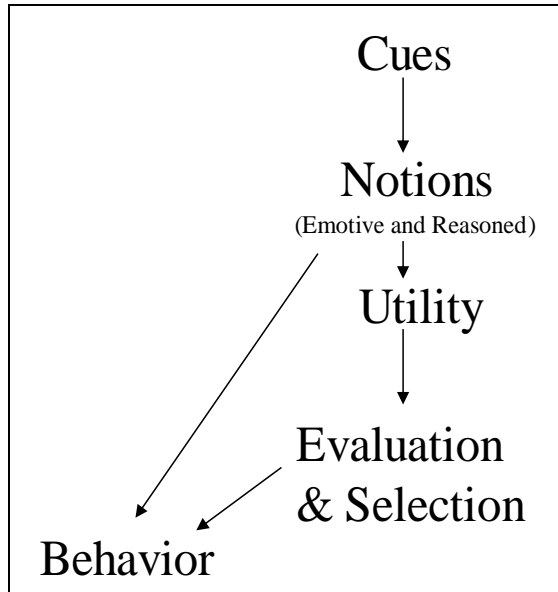


Figure 19: Transformation IV

The blueprint processes in the early phases of model usage do not take advantage of the expectation and incongruity capability of the short-mode use of the model. The section 2 above does indicate how an extension of the blueprint can be used to parameterize these added components. The long-mode (with learning) use of the model can also have its cognitive resource equations partially parameterized with the blueprint extensions. The addition of incongruity and expectation is illustrated in Figure 20. Other than for the absence of Cognitive Resources, Figure 20 corresponds to the actual information flows within the computational model.

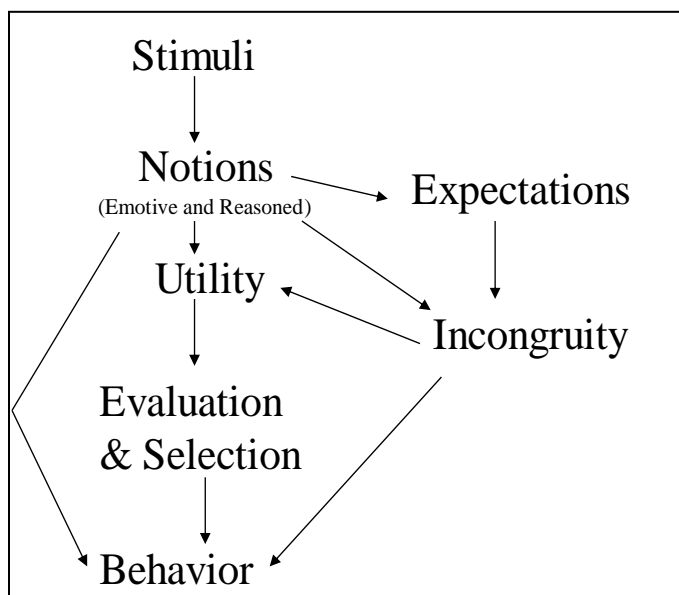


Figure 20: Transformation V



Figure 21 presents another way of looking at the blueprint that depicts all the components that must explicitly map into the computation framework. (See section 3 below.) The PAFF and NAFF are the positive and negative affects, respectively. The CBC noted in the figure is the cue activation rates in Figure 2. The determinants are just the beliefs affecting realized behavior as noted in Figure 15.

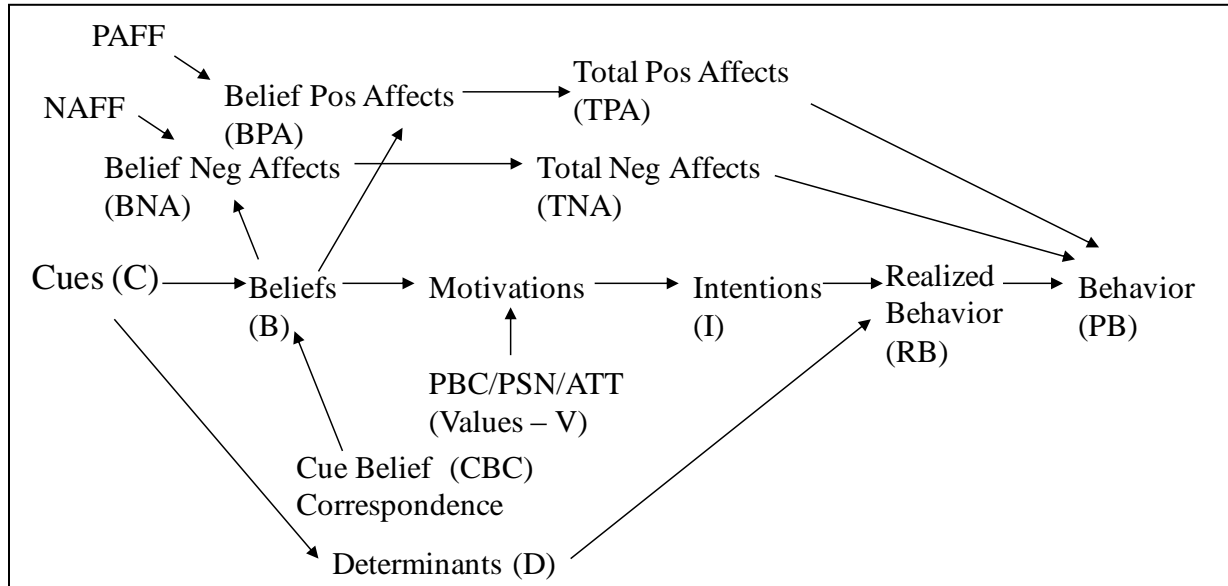


Figure 21: Information Flows.

Generically, Figures 20 and 21 reflect the underlying logic used in the next section to elaborate the conversion of blueprint information to model parameterization.

### 3. Parameterization Logic

This section describes a representatively-complicated example to show the algebra of the *direct* translation from the blueprint to the computational model. The pages below will explain the numeracy starting at the top of the Figure 22 and working down. The example only covers a single decision, but the replication of the process can incorporate any number of decisions.

The actual logic for the numeracy is determined by starting with the Behavior (PB) and establishing what has to be “calculated” to determine it – and then where the information is in the Blueprint.

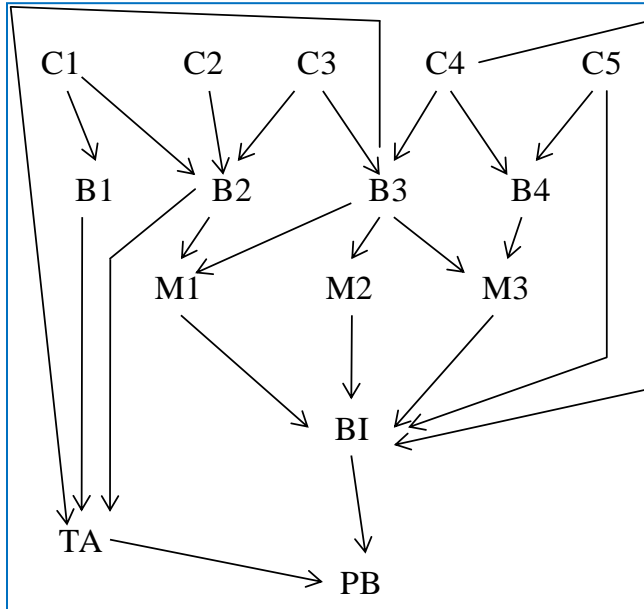


Figure 22: A Representative Blueprint (for one decision)

#### 3.1 Cues to Beliefs

This subsection considers the conversion of Cues to Beliefs. The mathematical terms are noted in the box below. Figure 23 show a blow-up of the concepts displayed in Figures 21 and 22.

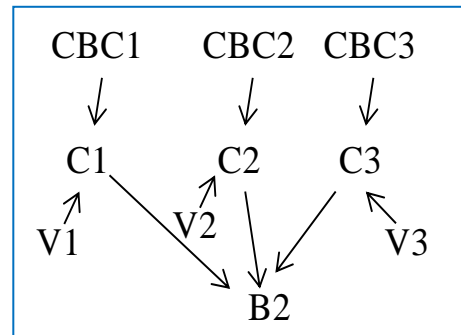
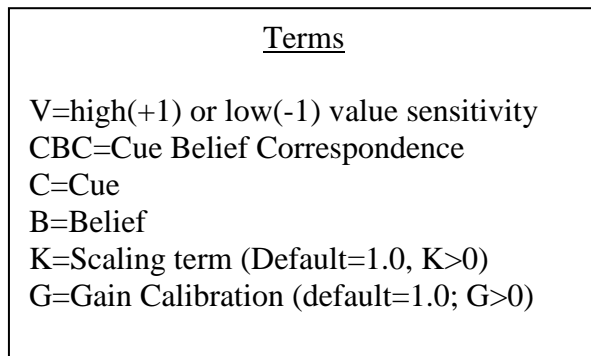


Figure 23: Cues to Beliefs

While in the blueprint, implicitly:

$$B_i = \sum(j)(C_j C_{ij} * C_j)$$

Or, equivalently in a rule-based context,

$$B = \text{True, if } C > \text{threshold}$$

in the model:

$$B_j = K_j * \text{Prod}(i)(C_i \wedge (G_j * V_i * C_j))$$

If the B is not an “OR” logic, then it can be separated into “f” beliefs or into a combination of “and” and “or” beliefs:

$$B_f = K_f * C_f \wedge (G_f * V_f * C_f)$$

If the “belief” is strictly an “AND” logic, then it needs to be separated in to multiple beliefs. In model, Belief is a Notion (P) based on Cues (S), and cognitive resource contributions(N):

$$P(r, y) = \alpha(r, y) * \prod_z S(r, z)^{\beta(r, y, z) * N(r, y, z)}$$

The variable N, representing the (attitude) impact of cognitive resources is assumed constant with a value of 1.0 for this discussion.

For entity “r”, notion “j” and cue” i”, the terms in the modes equations then become:

$$P(r, j) = B_j \quad \alpha(r, j) = K_j \quad \beta(r, j, i) = G_j * V_i * C_j C_i$$

### 3.2 Beliefs to Motivations

In the blueprint, beliefs are first noted at their “feature” or superficial level. Each of these beliefs is associated with deeper implications. For example, a superficial belief that there is a decrease in food availability may be associated with a deeper belief that there is a threat of starving.

Conversely, a superficial belief that there is an increase in food availability may be associated with an impression that “good times” have returned. If the deeper “decrease” belief is explicitly recognized, then the deeper “increase” belief may be that “there is a need to take advantage of this moment by hoarding.” In all cases, the actual belief is on a positive and negative continuum. That is, an actual increase may be seen as a decrease if the expectation was for a huge increase or that there is a need (winter is coming) for purchasing food to stock inventories.

Terms
M=Motivation
B=Belief
BP=Belief Potency (Default=1.0)

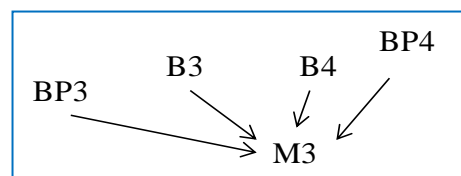


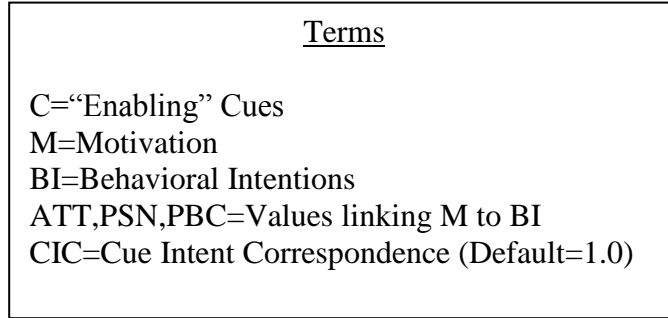
Figure 24: Beliefs to Motivation

BP needs to be added to blueprint for motivations in same way as CBC for behaviors.  
M is activated (primed) if any of the corresponding B are active.

$$M_i = \text{Sum}(j)(BP_{ji} * B_j)$$

### 3.3 Motivations to Intentions

This section describes the translation of motivations to intentions.



CIC needs to be added to the blueprint, same as CBC. This logic is shown in Figure 25.

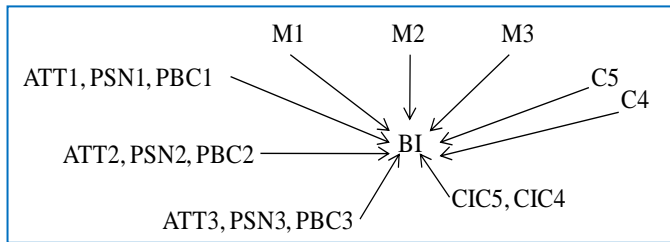


Figure 25: Motivation to Intent

In the model, the intention is a QCT (Qualitative Choice Theory) selection based on utility. The motivations are the utility of the selection, but in the blueprint are also modified by cues. The blueprint implies the utility “U” of choice “c”:

$$U_c = \text{Sum}(j)(ATT_j + PSN_j + PBC_j) * M_j * (CIC5 * C5 + CIC4 * C4)$$

or generally as:

$$U_c = \text{Sum}(j)((ATT_j + PSN_j + PBC_j) * M_j * \text{sum}(k)(CIC_k * C_k))$$

or equivalently as

$$U_c = \text{Sum}(j)(\text{sum}(i)(ATT_j + PSN_j + PBC_j) * M_j * (CIC_i * C_i))$$

We can make a new term  $D_{ji}$ :

$$D_{ji} = (ATT_j + PSN_j + PBC_j) * CIC_i$$

and a  $D(k)$  by noting

$$D(j*i) = D_{ji} \text{ where "k" is new } j*i \text{ set.}$$

Further, we can make a new equivalent belief (notion) set by noting again that:

$$M_g = \text{Sum}(h)(BP_{gh} * B_h)$$

and

$$B_h = \alpha * \text{prod}(i)(C_i \wedge CBC_{ih})$$

$$B_{ij} = B_i * C_j$$

with an implicit  $CBC_j$  of 1.0, is an equivalent new  $B_h = B(i*j)$  set. The  $U$  then becomes:

$$U_c = \text{Sum}(k)(\text{sum}(h)(D_k * BP_{kh} * B_h))$$

We can make a larger “l” set= $k*h$  and set:

$$\beta_l = \beta(k*h) = D_k * BP_{kh}.$$

$U$  then has the canonical form:

$$U_{rc} = \alpha_{rj} + \text{sum}(y)(\beta_{rcy} * B_{ry})$$

or more exactly (where  $P$  is the Notion of the belief “B”),

$$U_{rc} = \alpha_{rj} + \text{sum}(y)(\beta_{rcy} * N_{rc} * P_{ry}),$$

where  $N$  is defined previously.

$\alpha_{rj}$  = a calibrated constant (unless have data to parameterize)

$$\beta_{tji} = K * (ATT + PSN + PBC) * CIC$$

and the  $K$  is a (calibrated) scaling term because the units of measure and “reference values” for beliefs/notions ( $B/P$ ) are either calibrated (to intent) or defined with actual data (to actual experience). Note that we may want to treat “ $\text{sum}(k)(CIC_k * C_k)$ ” as a continuous belief, rather than as a discrete rule. Then there is either a new  $B$ :

$$B = \alpha * \text{prod}(k)(C_k \wedge CIC_k)$$

or this new B is combined with the old beliefs as noted early – now as  $\text{prod}(k)(C_k^{\wedge}CIC_k)$  rather than as individual  $C_k^{\wedge}1.0$ . Because Stalin could, for example, oppress opposition that wasn't there, “enabling” an intent may also be a perception. Therefore, the approach is to use the information in “CUE DETERMINANTS FOR BEHAVIORAL INTENTION SELECTION” in Figure 8 to make a separate belief for inclusion in the intent utility function.

In a binary choice, the actual intent calculation uses QCT as defined (and extended) in the main text of this document.

$$BI = e^U / (e^0 + e^U)$$

### 3.4 Negative and Positive Affect

This section considers how positive affect (PA) and negative affect (NA) determine the total affect (TA) as depicted in Figure 26.

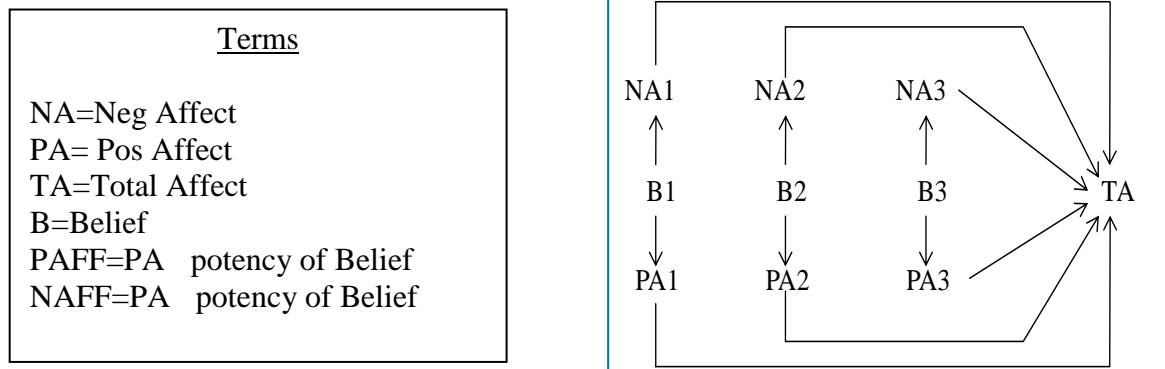


Figure 26: Positive and Negative Affect

The equations are a simple weighting of the Beliefs to make a total sum:

$$PA_{ij} = PAFF_{ij} * B_i$$

$$NA_{ij} = NAFF_{ij} * B_i$$

$$TA_j = \text{sum}(i)(PA_{ij} + NA_{ij})$$

### 3.5 Intentions to Behavior

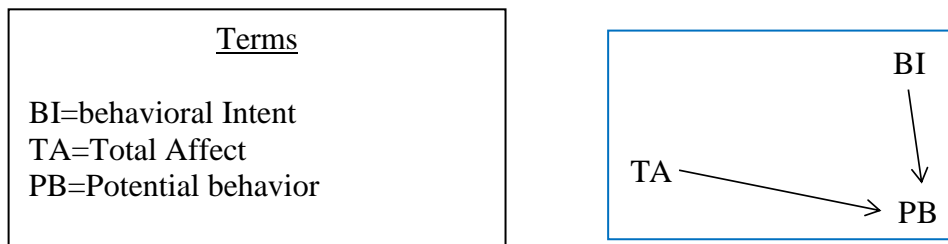


Figure 27: Intentions to Behavior

This section describes how to combine the BI and TA to calculate the actual behavior (PB). The value of BI will be between 0.0 and 1.0. The blueprint specifies the outcome of the intention (low, medium and high). As always, the blueprint is an under-specification, but contains indicative (Bayesian relevant) information.

Some arbitrary quantification of L/M/H could be:

L	M	H
<0.25	0.5	>0.75
<0.33	1.0	>3.0

“L” could be lower and “H” could be higher (but always >0).

There could also be “extreme condition” truths where it is “taken for granted” that a certain values of  $P_i$  makes  $e^U$  become negligible (e.g., 0.02) or “very” large (e.g., 20.0). These extremes can algebraically define the  $\alpha$  in:

$$U_j = \alpha_j + \sum(i)(\beta_{ij} * P_i).$$

Via truth table creation (See section 5 below), the  $\beta$  are algebraically defined (albeit often by trial and error, or by a simple automated-search routine, in the absence of sufficient data). Note the estimated  $\beta$  are not necessarily the derived ones because the  $P_i$  has to be normalized. To see this, think of the derived  $\beta$ , but then rewrite  $U$  as:

$$U_j = \alpha_j + \sum(i)(\beta_{ij} * (P_i / P_{0i})).$$

The unknown normalizations ( $P_{0i}$ ) are subsumed in the estimated  $\beta$ . Added data would allow the separate extraction of  $P_{0i}$ . One can see this logic in the physical procedure of the actual study model.

$$PB = BI * F(TA)$$

Conceptually, TA is meant to amplify BI. In the model,

$$PB = BI * e^U$$

where “ $e^U$ ” is the amplification and:

$$U = f(TA).$$

TA can be rewritten as:

$$TA_j = \sum(i)(PAFF_{ij} * B_i + NAFF_{ij} * B_i)$$

Or

$$TA_j = \sum(i)((PAFF_{ij} + NAFF_{ij}) * B_i)$$

or

$$TA_j = \sum(i)(\beta_{ij} * B_i),$$

where

$$\beta_{ij} = PAFF_{ij} + NAFF_{ij}$$

or by noting  $B_i = P_i$  in the model:

$$TA_j = \sum(i)(\beta_{ij} * P_i)$$

So in model terms,

$$U_j = \alpha_j + \sum(i)(\beta_{ij} * P_i)$$

#### 4. Example

This section illustrates a numerical example using the Blueprint in Figures 1 through 10. This logic is displayed in Figure 28.

##### 4.1 Cue to Belief Calculation

$B_x$ , in figure 28, is a belief formed from  $C_4$ ,  $C_6$  and  $C_9$  per “CUE DETERMINANTS FOR BEHAVIORAL INTENTION SELECTION” in Figure 8.  $C_4$  and  $C_6$  are negative,  $C_9$  is positive. In the absence of additional information:

$$B_x = \alpha_x * C_4^{(-0.33 * \beta_x)} * C_6^{(-0.33 * \beta_x)} * C_9^{(0.34 * \beta_x)}.$$

Manual calibration could start with  $\alpha_x = 1.0$ ;  $\beta_x = 1.0$



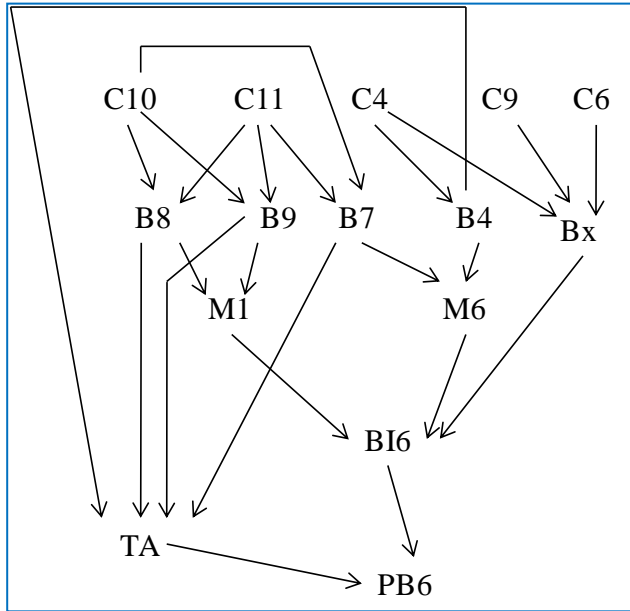


Figure 28: An Example for Calculation

B8 and B9 both use C10(-80),C11(-20):

$$B8 = \alpha_8 * C10^{(-0.8 * \beta_8)} * C11^{(-0.2 * \beta_8)}$$

$$B9 = \alpha_9 * C10^{(-0.8 * \beta_9)} * C11^{(-0.2 * \beta_9)}$$

B7 uses C10(+80),C11(+20):

$$B7 = \alpha_7 * C10^{(+0.8 * \beta_7)} * C11^{(+0.2 * \beta_7)}$$

B4 uses C4(-100).

$$B4 = \alpha_4 * C10^{(1.0 * \beta_4)}$$

Manual calibration could start with  $\alpha_{\#}=1.0$ ;  $\beta_{\#}=1.0$

#### 4.2 Belief to Motivation Calculation

In the context of section 3.2 and Figure 28, the Motivations are:

$$M1 = \gamma_1 * B8 + \delta_1 * B9$$

$$M6 = \gamma_6 * B7 + \delta_6 * B4$$

In the absence of added information, manual calibration could start with  $\gamma_{\#}=1.0$ ;  $\delta_{\#}=1.0$

### 4.3 Motivations to Intentions Calculation

By just using the M# as utility terms, the Utility to calculate BI6 is then:

$$U_6 = \eta_6 + \gamma_1 * B_8 + \delta_1 * B_9 + \gamma_6 * B_7 + \delta_6 * B_4 + \lambda * B_x$$

In the absence of further information, B9 and B8 are indistinguishable in calibration terms, so we might as well reduce dimensionality (for calibration – we back out a “redundant” B ex post), so use the equivalent:

$$U_6 = \eta_6 + \gamma_1 * B_8 + \gamma_6 * B_7 + \delta_6 * B_4 + \lambda * B_x$$

At this point we are only solving for BI6 so, we can use a truth table to determine when the calibration matches the intended logic. See Section 5 for an example of this process.

Once the calibration meets is consistent with what is intended by the SME, per the blueprint content, the SME can verify it by exercising the calibrated code.

### 4.4 Intentions to Behavior Calculation

TA capture the amplification (or triggering of the intention) of behavior. A simple additive utility logic for this is:

$$U = \sigma + v * TA$$

Remember:

$$\beta_{ij} = PAFF_{ij} + NAFF_{ij}$$

and noting in the model that:

$$B_i = P_i$$

and that

$$TA_j = \sum(i) (\beta_{ij} * B_i)$$

From Figure 5:

	PAFF	NAFF
B8	0	10
B9	1	8
B7	8	0
B4	0	10

Thus:

$$UA = \sigma + v * \sum(i)(\beta_{ij} * B_i)$$

The amplification is  $e^U$  with the behavior  $r(PB6)$  as:

$$PB6 = BI6 * e^{UA}$$

Based on the quantification of smaller (low) versus larger (high) for PB6 [e.g., assume,  $L/M/H = <0.33/1.0/>3.0$ ] Using the BI6 results, calibrate  $\sigma$  and  $v$ , as demonstrated in Section 5. To obtain small values of amplification,  $\alpha \ll 0$ .

Once the calibration meets is consistent with what is intended by the SME, per the blueprint content, the SME can verify it by exercising the calibrated code.

## 5. Long-Term Processes

Section 4 describes behaviors in the absence of learning. It is essentially a static representation. The calculational model allows learning and fully dynamic simulations. This section describes how data within the blueprint can be used to determine dynamic parameterizations.

### 5.1 Notional Frequency and Recency

Recency recognizes the lingering affect of Notions. The typical recency ( $R$ ) is the time since the last typical cues (causing a notion of typical magnitude  $V$ ). In figure 13, the Psychological Magnitude ( $PM$ ) is the ratio between the current assimilated level of the notion compared to its initial (peak) value. The  $R$  and  $PM$  can determine the time constant ( $\tau_a$ ) for assimilation.

$$\tau_a = R / \ln(PM)$$

In Figure 11, Internal Incongruity ( $IC$ ) is the same concept as  $RM$  except applied to for cognitive resources. “Duration of Current Environment” ( $T$ ) in Figure 12 is the length of time over which the averaging of the frequency corresponds. Frequency is the number of times the cues occur within the Applicable Time Scale. The frequency,  $V$ , and  $IC$  can be used to determine the value of the cognitive resource and the time constant on learning. This logic assumes an estimate of the average effective value ( $\psi$ ) of  $V$  based on recency and frequency, as explained in the next section:

$$CR = V * (1 - \exp(\psi * T / \tau_c)); \tau_c = \psi * T / \ln(1.0 - IC)$$

$DIFF$ , in Figure 13 is the incongruity between the current notion and its expected value. With an assumption of offset value ( $\sigma$ ),  $DIFF$  can be used with the above information to estimate the time constant for expectation formation ( $\tau_H$ ) and the initial value of the expectations ( $H$  – here under “remembered” assumptions and less than  $V$ ).

$$H=V/(|Diff|+(1+\sigma))$$

$$\tau H=R/\ln(1-H/V)$$

## 5.2 Average Notion

Combining the concept of frequency “f” from Figure 11) and recency from Figure 13 allows an estimate of the average notion, which is also the estimated value of expectations (H).

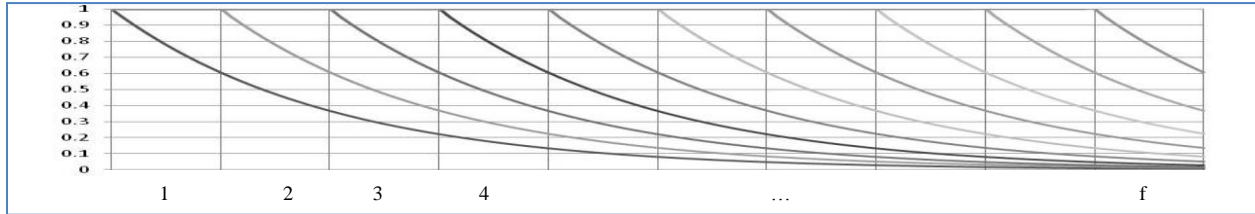


Figure 29: Remembrance of Recurrent Recency

The maximum value in Figure 29 is the “being remembered” value and its average is the integral over the  $1/f$  time-period. In other words, based on an assumption that event occur over a short timeframe relative to the frequency (f) of the occurrence, and that the recency time ( $\tau$ ) reflects the lingering signature of the notional impact of the event, then the average notion is:

$$\psi = \int_0^{1/f} e^{-t/\tau_R} * dt / \int_0^{1/f} dt = f * \tau_R * (1 - e^{-1/(\tau_R * f)})$$

## 5.3 Behavioral Frequency and Recency

This Behavioral Frequency and Recency information is actually representative of the feedback, where the entity’s behavior becomes a remembered, self-referencing cue (corresponding to a new input belief have associated frequency and recency characterizations). As such, a second tier of blueprints would include the entities PB (behavior) as an input cue that affects the downstream beliefs, motivations, intentions and the intensity of subsequent behaviors.

## 6. Rule-based QCT

This section provides two examples of using a truth-table inherent in the Blueprint to estimate parameters consistent with the SME representation in the Blueprint. A truth table sets the Cues to all possibilities of “on” (1.0) or “off” (0.0) -- with the SME (possibly with the help of an analysts) then applying the truth table logic as if it were deterministic and rule-based to determine the resulting behavior and intermediate components. “Rule-based” is meant to imply if-then logic. In

the actual model, the results are based on a probabilistic response. This extension is discussed in section 7 below.

The column with black-type headings in Figure 30 show the information as implied for the blueprint. The red-type columns show the modeled values. The simulated values are set to unity if the calculated probability is greater than 50% and to 0.0 if less than 50%.

C44	C71	C70	B1	B16	Neg Affect	Pos Affect	M1	M10	M11	B127	Calc Intent	PB44	PB45	PB46	Calc Low	Calc Med	Calc Hi
0	0	0	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	1	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	0	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	1	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	0	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	1	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	0	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	1	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	0	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	0	1	0	0	0.0	0.0				0	0	0	0	0	0	0	0
0	1	0	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	1	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	0	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	1	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	0	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	1	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	0	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
0	1	1	0	1	0.9	0.0		primed	primed+selected	1	1	1	0	1	0	0	1
1	0	0	1	0	0.7	0.3	primed+selected			0 (but primed)	0	0	0	0	0	0	0
1	0	1	1	0	0.7	0.3	primed+selected			1	1	1	0	0	0	1	0
1	0	0	1	0	0.7	0.3	primed+selected			0 (but primed)	0	0	0	0	0	0	0
1	0	1	1	0	0.7	0.3	primed+selected			1	1	1	0	0	0	1	0
1	0	0	1	0	0.7	0.3	primed+selected			0 (but primed)	0	0	0	0	0	0	0
1	0	1	1	0	0.7	0.3	primed+selected			1	1	1	0	0	0	1	0
1	0	0	1	0	0.7	0.3	primed+selected			0 (but primed)	0	0	0	0	0	0	0
1	0	1	1	0	0.7	0.3	primed+selected			1	1	1	0	0	0	1	0
1	1	0	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	1	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	0	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	1	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	0	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	1	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	0	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	1	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	0	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1
1	1	1	1	1	0.9	0.3	primed+selected	primed	primed	1	1	1	0	1	0	0	1

Figure 30: First Example Truth Table.

	a0	b1	b2	b3	
intent	-2.99573	4	1	4	
	low prob				
$U=a_0+b_1*P_1+b_2*P_2+b_3*P_3$					
	a0	b1	b2	b3	b4
intensity	-1.60944	0.5	2	0.5	-0.5
	low amplification				
$U=a_0+b_1*P_5+b_2*P_1+b_3*P_6+b_4*p_4$					
Perception (notion)			with unity exponent		
P1	C71			C71	
P2	$C44^{0.5}*C71^{0.5}$			$C44*C71$	
P3	$C44^{0.5}*C70^{0.5}$			$C44*C70$	
p4	$C44^{.33}*C71^{.33}*C70^{.34}$			$C44*C71*C70$	
p5	C44			C44	
P6	C70			C70	

Figure 31: First Example Calculations

The mathematical solution is essentially composed of two utilities; one determines the probability of the intent and the other the intensity of the behavior. The top portion of Figure 31 shows the in the values of the parameters, with the  $\alpha$  set to generate a low probability of occurrence in the absence of any cues. The middle portion shows the intensity, with the  $\alpha$  set to generate a low intensity in the absence of any cues. The lower portion shows the notion (belief) terms. Figure 32 shows the calculated values of the intent utility, the actual probability, and the amplification (intensity).

Utility	Intent Prob	Amplification
-3.00	0.05	0.20
-3.00	0.05	0.33
-3.00	0.05	0.20
-3.00	0.05	0.33
-3.00	0.05	0.20
-3.00	0.05	0.33
-3.00	0.05	0.20
-3.00	0.05	0.33
1.00	0.73	1.48
1.00	0.73	2.44
1.00	0.73	1.48
1.00	0.73	2.44
1.00	0.73	1.48
1.00	0.73	2.44
1.00	0.73	1.48
1.00	0.73	2.44
-3.00	0.05	0.33
1.00	0.73	0.54
-3.00	0.05	0.33
1.00	0.73	0.54
-3.00	0.05	0.33
1.00	0.73	0.54
-3.00	0.05	0.33
1.00	0.73	0.54
2.00	0.88	2.44
6.00	1.00	2.44
2.00	0.88	2.44
6.00	1.00	2.44
2.00	0.88	2.44
6.00	1.00	2.44
2.00	0.88	2.44
6.00	1.00	2.44

Figure 32: First Example Calculated Values

In a second example, more cues interact with motivation and behaviors in a more complicated fashion. All tables have the same interpretation as in the previous examples. In all instance, note that the mathematical translation exactly matches the SME “truth table” results.

C9	C44	C46	C47	B1	Neg Affect	Pos Affect	M1	BI23	Calc Intent	PB30 (Low)	PB34 (Hi)	PB39 (Hi)	Calc Low	Calc Hi
0	0	0	0	0	0.0	0.0		0	0	0	0	0	0	0
1	0	0	0	1	0.0	0.0	primed+selected	0	0	0	0	0	0	0
0	0	0	1	0	0.0	0.0		0	0	0	0	0	0	0
1	0	0	1	1	0.0	0.0	primed+selected	1	1	0	1	1	0	1
0	0	1	0	0	0.0	0.0		0	0	0	0	0	0	0
1	0	1	0	1	0.0	0.0	primed+selected	1	1	0	1	1	0	1
0	0	1	1	0	0.0	0.0		0	0	0	0	0	0	0
1	0	1	1	1	0.0	0.0	primed+selected	1	1	0	1	1	0	1
0	0	0	0	0	0.9	0.0		0	0	0	0	0	0	0
1	0	0	0	1	0.9	0.0	primed+selected	0	0	0	0	0	0	0
0	0	0	1	0	0.9	0.0		0	0	0	0	0	0	0
1	0	0	1	1	0.9	0.0	primed+selected	1	1	0	1	1	0	1
0	0	1	0	0	0.9	0.0		0	0	0	0	0	0	0
1	0	1	0	1	0.9	0.0	primed+selected	1	1	0	1	1	0	1
0	0	1	1	0	0.9	0.0		0	0	0	0	0	0	0
1	0	1	1	1	0.9	0.0	primed+selected	1	1	0	1	1	0	1
0	1	0	0	1	0.7	0.3	primed+selected	0 (but primed)	0	0	0	0	0	0
1	1	0	0	1	0.7	0.3	primed+selected	0 (but primed)	0	0	0	0	0	0
0	1	0	1	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
1	1	0	1	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
0	1	1	0	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
1	1	1	0	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
0	1	1	1	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
1	1	1	1	1	0.7	0.3	primed+selected	1	1	0	1	1	0	1
0	1	0	0	1	0.9	0.3	primed+selected	0 (but primed)	0	0	0	0	0	0
1	1	0	0	1	0.9	0.3	primed+selected	0 (but primed)	0	0	0	0	0	0
0	1	0	1	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1
1	1	0	1	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1
0	1	1	0	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1
1	1	1	0	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1
0	1	1	1	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1
1	1	1	1	1	0.9	0.3	primed+selected	1	1	0	1	1	0	1

Figure 33: Second Example Truth Table

	a0	b1	b2	b3	b4
intent	-2.99573	3	3	3	3
low prob					
$U=a0+b1*P1+b2*P2+b3*P3+b4*P4$					
	a0	b1	b2	b3	
intensity	-2.99573	3	3	-2	
low amplification					
$U=a0+b1*P5+b2*P6+b3*P7$					
Perception (notion)			With unity exponent		
P1	$C9^{0.5}*C46^{0.5}$		$C9*C46$		
P2	$C9^{0.5}*C47^{0.5}$		$C9*C47$		
P3	$C44^{0.5}*C46^{0.5}$		$C44*C46$		
P4	$C44^{0.5}*C47^{0.5}$		$C44*C47$		
P5	C9		C9		
P6	C44		C44		
P7	$C9^{0.5}*C44^{0.5}$		$C9*C44$		

Figure 34: Second Example Calculations

Utility	Intent Prob	Amplification
-3.00	0.05	0.05
-3.00	0.05	1.00
-3.00	0.05	0.05
0.00	0.50	1.00
-3.00	0.05	0.05
0.00	0.50	1.00
-3.00	0.05	0.05
3.00	0.95	1.00
-3.00	0.05	0.05
-3.00	0.05	1.00
-3.00	0.05	0.05
0.00	0.50	1.00
-3.00	0.05	0.05
0.00	0.50	1.00
-3.00	0.05	0.05
3.00	0.95	1.00
-3.00	0.05	1.00
-3.00	0.05	2.73
0.00	0.50	1.00
3.00	0.95	2.73
0.00	0.50	1.00
3.00	0.95	2.73
3.00	0.95	1.00
9.00	1.00	2.73
-3.00	0.05	1.00
-3.00	0.05	2.73
0.00	0.50	1.00
3.00	0.95	2.73
0.00	0.50	1.00
3.00	0.95	2.73
3.00	0.95	1.00
9.00	1.00	2.73

Figure 35: Second Example Calculated Values



## 7. Probabilistic QCT

This section considers how to interpret rule-based (if-then) logic with QCT. The example is based on a hypothesized SME stating that a belief is evoked when the cue C4 has a value below 0.3 and a cue C8 has a value greater than 0.3 [i.e.,  $(C4 < 0.3) \text{ AND } (C8 > 0.3)$ ] We look at three examples of implementing this statement in probabilistic mathematics. The first is to simply take the statement at face value as a notion “P” (belief) taking the form :

$$P = \alpha * C4^{\beta_4} * C8^{\beta_8}$$

The intent and behavior is only a function of this notion and, for a binary choice, has the utility U with the form:

$$U = \alpha + \beta * P$$

Figure 36 shows the values of the belief as a function of the cues and Figure 38 shows the surface generated. Figures 37 and 39 shows the consequential intent/behavior that reflects the probabilistic representation of the  $C4 < 0.3 \text{ AND } C8 > 0.3$  rule.

Intensity of Belief																			
Cues		C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8
		Values	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5
C4	0.1	1.116	1.234	1.340	1.436	1.525	1.608	1.686	1.760	1.830	1.897	1.961	2.023	2.083	2.140	2.196	2.250	2.303	
C4	0.125	0.987	1.091	1.185	1.270	1.349	1.422	1.491	1.556	1.619	1.678	1.735	1.789	1.842	1.893	1.942	1.990	2.037	
C4	0.15	0.893	0.987	1.072	1.149	1.220	1.286	1.349	1.408	1.464	1.518	1.569	1.619	1.666	1.713	1.757	1.800	1.842	
C4	0.175	0.820	0.907	0.985	1.055	1.121	1.182	1.239	1.293	1.345	1.394	1.442	1.487	1.531	1.573	1.614	1.654	1.693	
C4	0.2	0.762	0.843	0.915	0.981	1.041	1.098	1.151	1.202	1.250	1.296	1.340	1.382	1.423	1.462	1.500	1.537	1.573	
C4	0.225	0.715	0.790	0.858	0.919	0.976	1.029	1.079	1.126	1.171	1.214	1.256	1.295	1.333	1.370	1.406	1.441	1.474	
C4	0.25	0.674	0.746	0.809	0.867	0.921	0.971	1.018	1.063	1.105	1.146	1.185	1.222	1.258	1.293	1.327	1.359	1.391	
C4	0.275	0.640	0.707	0.768	0.823	0.874	0.922	0.966	1.009	1.049	1.087	1.124	1.160	1.194	1.227	1.259	1.290	1.320	
C4	0.3	0.610	0.674	0.732	0.785	0.833	0.879	0.921	0.962	1.000	1.037	1.072	1.106	1.138	1.170	1.200	1.230	1.258	
C4	0.325	0.584	0.645	0.701	0.751	0.797	0.841	0.882	0.920	0.957	0.992	1.026	1.058	1.089	1.119	1.148	1.177	1.204	
C4	0.35	0.560	0.620	0.673	0.721	0.765	0.807	0.846	0.883	0.919	0.952	0.985	1.016	1.046	1.075	1.103	1.130	1.156	
C4	0.375	0.540	0.596	0.647	0.694	0.737	0.777	0.815	0.851	0.885	0.917	0.948	0.978	1.007	1.035	1.062	1.088	1.113	
C4	0.4	0.521	0.576	0.625	0.670	0.711	0.750	0.786	0.821	0.854	0.885	0.915	0.944	0.972	0.999	1.025	1.050	1.074	
C4	0.425	0.504	0.557	0.604	0.648	0.688	0.725	0.761	0.794	0.826	0.856	0.885	0.913	0.940	0.966	0.991	1.015	1.039	
C4	0.45	0.488	0.540	0.586	0.628	0.667	0.703	0.737	0.769	0.800	0.829	0.858	0.885	0.911	0.936	0.960	0.984	1.007	
C4	0.475	0.474	0.524	0.569	0.609	0.647	0.682	0.715	0.747	0.777	0.805	0.832	0.859	0.884	0.908	0.932	0.955	0.977	
C4	0.5	0.461	0.509	0.553	0.592	0.629	0.663	0.696	0.726	0.755	0.783	0.809	0.835	0.859	0.883	0.906	0.929	0.950	

Figure 36: Belief from Cues

Pseudo-Probability Behavior is Activated																			
Cue		C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8
	Value	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5	
C4	0.1	0.64	0.76	0.85	0.90	0.93	0.95	0.97	0.98	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	
C4	0.125	0.48	0.61	0.72	0.79	0.85	0.89	0.92	0.94	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	
C4	0.15	0.37	0.48	0.59	0.68	0.75	0.81	0.85	0.88	0.91	0.93	0.95	0.96	0.97	0.97	0.98	0.98	0.99	
C4	0.175	0.29	0.39	0.48	0.57	0.65	0.71	0.77	0.81	0.85	0.88	0.90	0.92	0.93	0.95	0.96	0.96	0.97	
C4	0.2	0.23	0.31	0.40	0.48	0.55	0.62	0.68	0.73	0.78	0.81	0.85	0.87	0.89	0.91	0.92	0.94	0.95	
C4	0.225	0.19	0.26	0.33	0.40	0.47	0.54	0.60	0.65	0.70	0.74	0.78	0.81	0.84	0.86	0.88	0.90	0.91	
C4	0.25	0.16	0.22	0.28	0.34	0.40	0.46	0.52	0.58	0.63	0.67	0.72	0.75	0.78	0.81	0.84	0.86	0.88	
C4	0.275	0.14	0.19	0.24	0.29	0.35	0.40	0.46	0.51	0.56	0.61	0.65	0.69	0.73	0.76	0.78	0.81	0.83	
C4	0.3	0.12	0.16	0.21	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.59	0.63	0.67	0.70	0.73	0.76	0.78	
C4	0.325	0.11	0.15	0.18	0.22	0.27	0.31	0.36	0.40	0.45	0.49	0.53	0.57	0.61	0.64	0.68	0.71	0.74	
C4	0.35	0.10	0.13	0.16	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52	0.56	0.59	0.63	0.66	0.69	
C4	0.375	0.09	0.12	0.15	0.18	0.21	0.25	0.28	0.32	0.36	0.40	0.44	0.47	0.51	0.54	0.58	0.61	0.64	
C4	0.4	0.08	0.11	0.13	0.16	0.19	0.22	0.26	0.29	0.32	0.36	0.40	0.43	0.46	0.50	0.53	0.56	0.59	
C4	0.425	0.08	0.10	0.12	0.15	0.17	0.20	0.23	0.26	0.29	0.33	0.36	0.39	0.43	0.46	0.49	0.52	0.55	
C4	0.45	0.07	0.09	0.11	0.13	0.16	0.18	0.21	0.24	0.27	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51	
C4	0.475	0.07	0.08	0.10	0.12	0.15	0.17	0.19	0.22	0.25	0.27	0.30	0.33	0.36	0.39	0.42	0.44	0.47	
C4	0.5	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.23	0.25	0.28	0.30	0.33	0.36	0.38	0.41	0.44	

Figure 37: Intent/Behavior from Single Composite Belief (Function of Cue input)

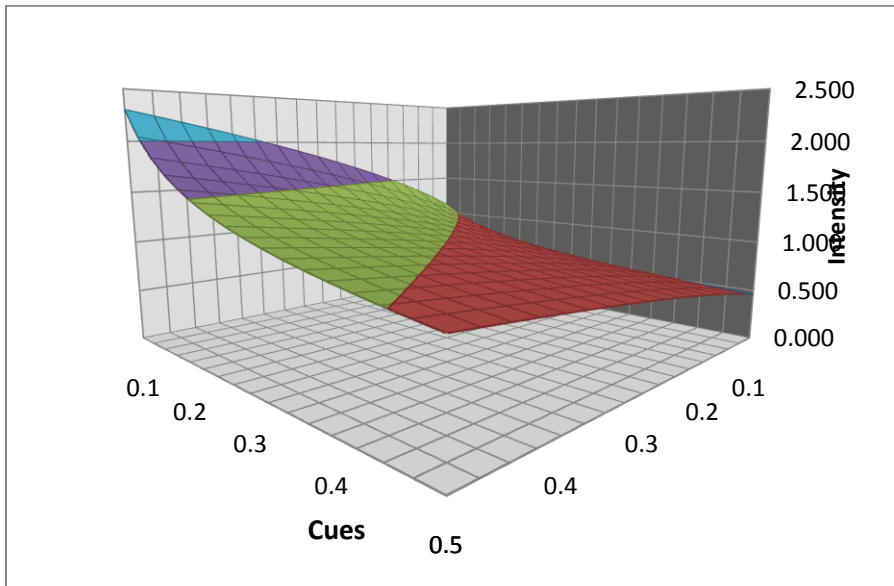


Figure 38: Belief Surface as a function of Cues.

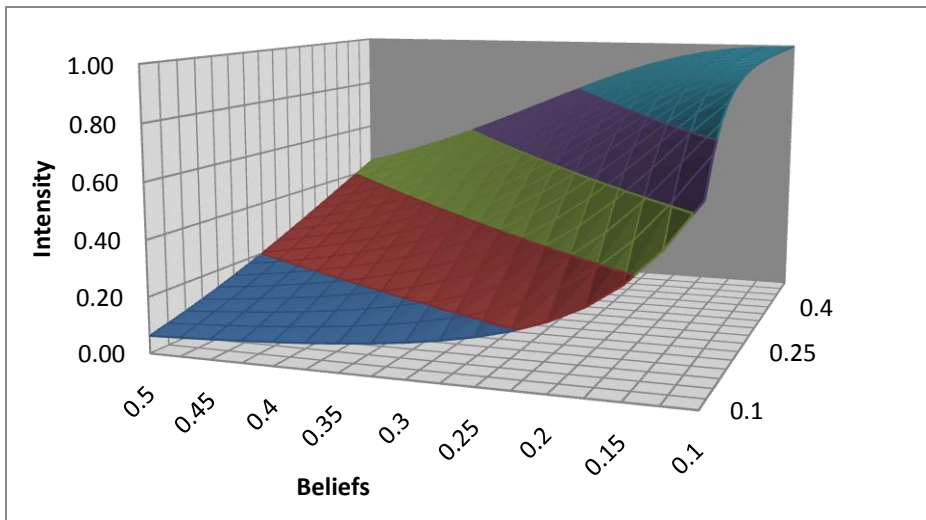


Figure 39: Intent/Behavior Approximation of Rule with Composite Belief

Figures 40 and 41 show the results of an example where each cue is treated as a separate belief

$$B1 = \alpha_1 * C_4$$

$$B2 = \alpha_2 * C_8$$

And each of these then become part of the utility function for the intent probability.

$$U = \alpha + \beta_1 * B1 + \beta_2 * B2$$

The blocked region in Figure 40 shows the area of the representing the Boolean truth values (value =1) for the rule [(C4<0.3) AND (C8>0.3)]. Figure 41 shows the approximation as a surface.

Probability Behavior is Activated																			
Cues		C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8
	Values	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5	
C4	0.1	0.00	0.00	0.73	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.125	0.00	0.00	0.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.15	0.00	0.00	0.00	0.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.175	0.00	0.00	0.00	0.00	0.76	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.2	0.00	0.00	0.00	0.00	0.17	0.86	0.97	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.225	0.00	0.00	0.00	0.00	0.03	0.47	0.84	0.93	0.96	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98	
C4	0.25	0.00	0.00	0.00	0.00	0.01	0.19	0.57	0.77	0.85	0.89	0.91	0.92	0.93	0.93	0.93	0.93	0.94	
C4	0.275	0.00	0.00	0.00	0.00	0.00	0.08	0.33	0.55	0.68	0.75	0.79	0.81	0.82	0.83	0.84	0.84	0.84	
C4	0.3	0.00	0.00	0.00	0.00	0.00	0.04	0.18	0.37	0.50	0.58	0.63	0.66	0.68	0.70	0.70	0.71	0.72	
C4	0.325	0.00	0.00	0.00	0.00	0.00	0.02	0.11	0.24	0.36	0.43	0.49	0.52	0.54	0.56	0.57	0.58	0.58	
C4	0.35	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.17	0.26	0.32	0.37	0.40	0.43	0.44	0.45	0.46	0.47	
C4	0.375	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.12	0.19	0.25	0.29	0.32	0.34	0.35	0.36	0.37	0.37	
C4	0.4	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.09	0.15	0.19	0.23	0.25	0.27	0.28	0.29	0.30	0.30	
C4	0.425	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.12	0.16	0.19	0.21	0.22	0.23	0.24	0.25	0.25	
C4	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.10	0.13	0.16	0.18	0.19	0.20	0.21	0.21	0.21	
C4	0.475	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.08	0.11	0.13	0.15	0.16	0.17	0.18	0.18	0.19	
C4	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.07	0.10	0.12	0.13	0.14	0.15	0.16	0.16	0.16	

Figure 40: Intent/Behavior from Dual Simple Beliefs

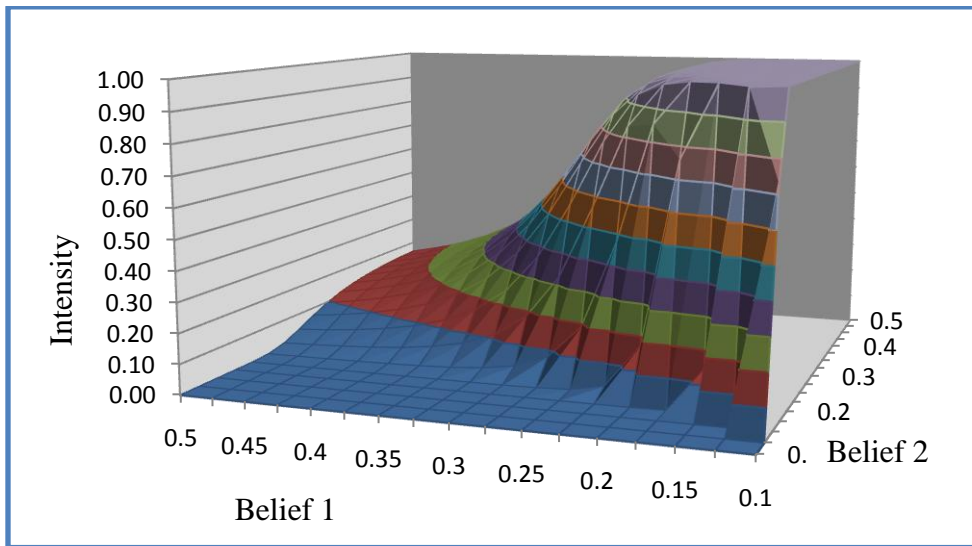


Figure 41: Intent/Behavior Surface from Dual Simple Beliefs

Lastly, Figures 42 and 43 show the outcome when the logic is portrayed as two composite beliefs both composed of C4 and C8. From Figure 43, it is clear the result does produce the expectation of a how a probabilistic version of the rule would look.

Probability Behavior is Activated																			
Cues		C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8	C8
	Values	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275	0.3	0.325	0.35	0.375	0.4	0.425	0.45	0.475	0.5	
C4	0.1	0.02	0.02	0.03	0.06	0.16	0.56	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.125	0.02	0.02	0.03	0.05	0.11	0.35	0.85	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.15	0.02	0.02	0.03	0.04	0.08	0.23	0.67	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.175	0.02	0.02	0.02	0.03	0.06	0.16	0.50	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.2	0.02	0.02	0.02	0.03	0.05	0.12	0.35	0.81	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.225	0.02	0.02	0.02	0.03	0.05	0.09	0.25	0.66	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.25	0.02	0.02	0.02	0.03	0.04	0.07	0.18	0.49	0.89	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.275	0.02	0.02	0.02	0.03	0.04	0.06	0.13	0.33	0.73	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.3	0.02	0.02	0.02	0.02	0.03	0.05	0.09	0.20	0.50	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	
C4	0.325	0.02	0.02	0.02	0.02	0.03	0.04	0.06	0.12	0.27	0.58	0.88	0.99	1.00	1.00	1.00	1.00	1.00	
C4	0.35	0.02	0.02	0.02	0.02	0.02	0.03	0.04	0.07	0.12	0.22	0.43	0.70	0.90	0.98	1.00	1.00	1.00	
C4	0.375	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.06	0.06	0.04	0.02	0.01	0.00	
C4	0.4	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
C4	0.425	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
C4	0.45	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
C4	0.475	0.02	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
C4	0.5	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

Figure 42: Intent/Behavior from Dual Composite beliefs.

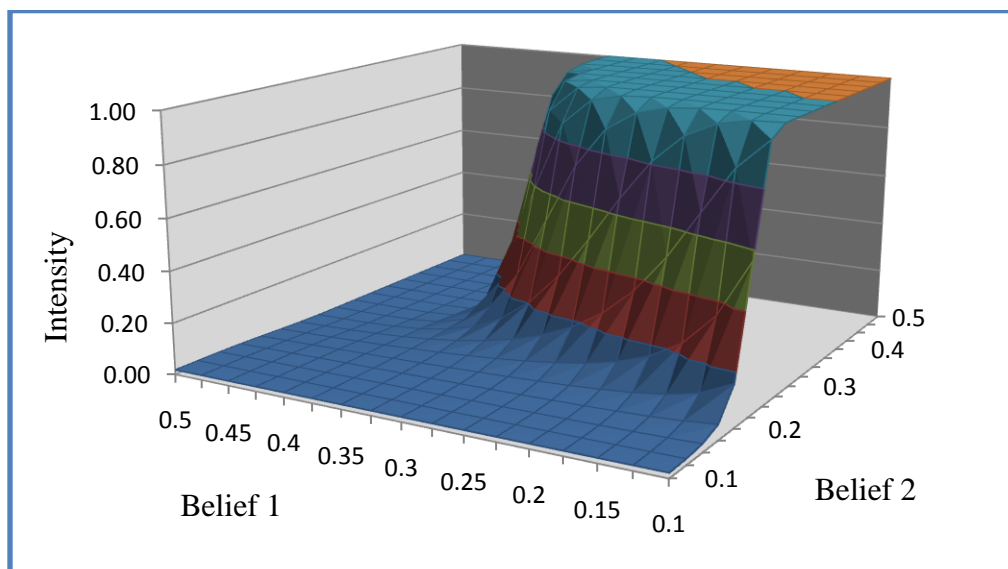


Figure 41: Intent/Behavior Surface from Dual Composite Beliefs

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